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Simulation Examples in LabVIEW

Hans-Petter Halvorsen



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Differential Equations



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Differential Equations

A general continuous differential equation can be written on this general form:

$$\frac{dx}{dt} = f(t, x)$$

A differential equation or a set of differential equations describes the dynamic behavior of a system

Differential Equations

Differential Equation on general form:

$$\frac{dy}{dt} = f(t, y), \quad y(t_0) = y_0$$

Initial condition



Different notation is used:

$$\frac{dy}{dt} = y' = \dot{y}$$

Example:

$$\frac{dy}{dt} = 3y + 2, \quad y(t_0) = 0$$

Initial condition

Differential equation

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Simulation Examples

1.order System



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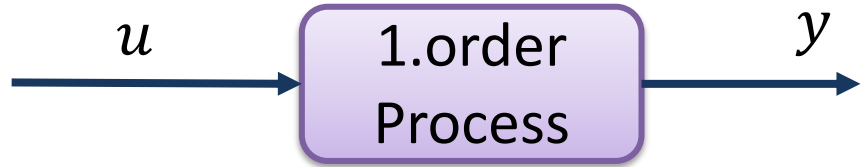
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1. Order System

Differential Equation of a 1. order System:

$$\dot{x} = -ax + bu$$

$$y = x$$



For control systems, u is typically the control value that comes from the controller, e.g., a PID controller.

Typically, in general we use x for internal variables in the process and y for the measured output(s). For larger systems we can have multiple x ($x_1, x_2 \dots$) and multiple y ($y_1, y_2 \dots$).

To simulate this model in LabVIEW you can e.g., make a discrete version of the model

1. Order System

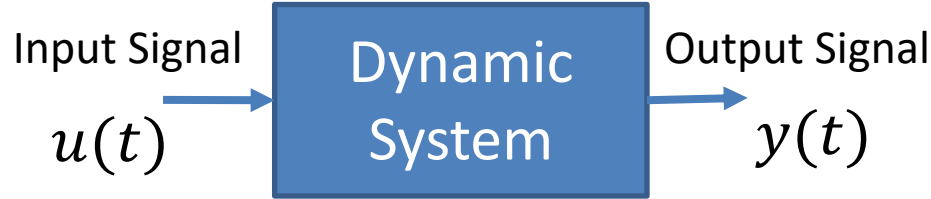
Assume the following general **Differential Equation**:

$$\dot{y} = -ay + bu$$

or:

$$\dot{y} = \frac{1}{T}(-y + Ku)$$

$$\text{Where } a = \frac{1}{T} \text{ and } b = \frac{K}{T}$$



Where K is the Gain and T is the Time constant

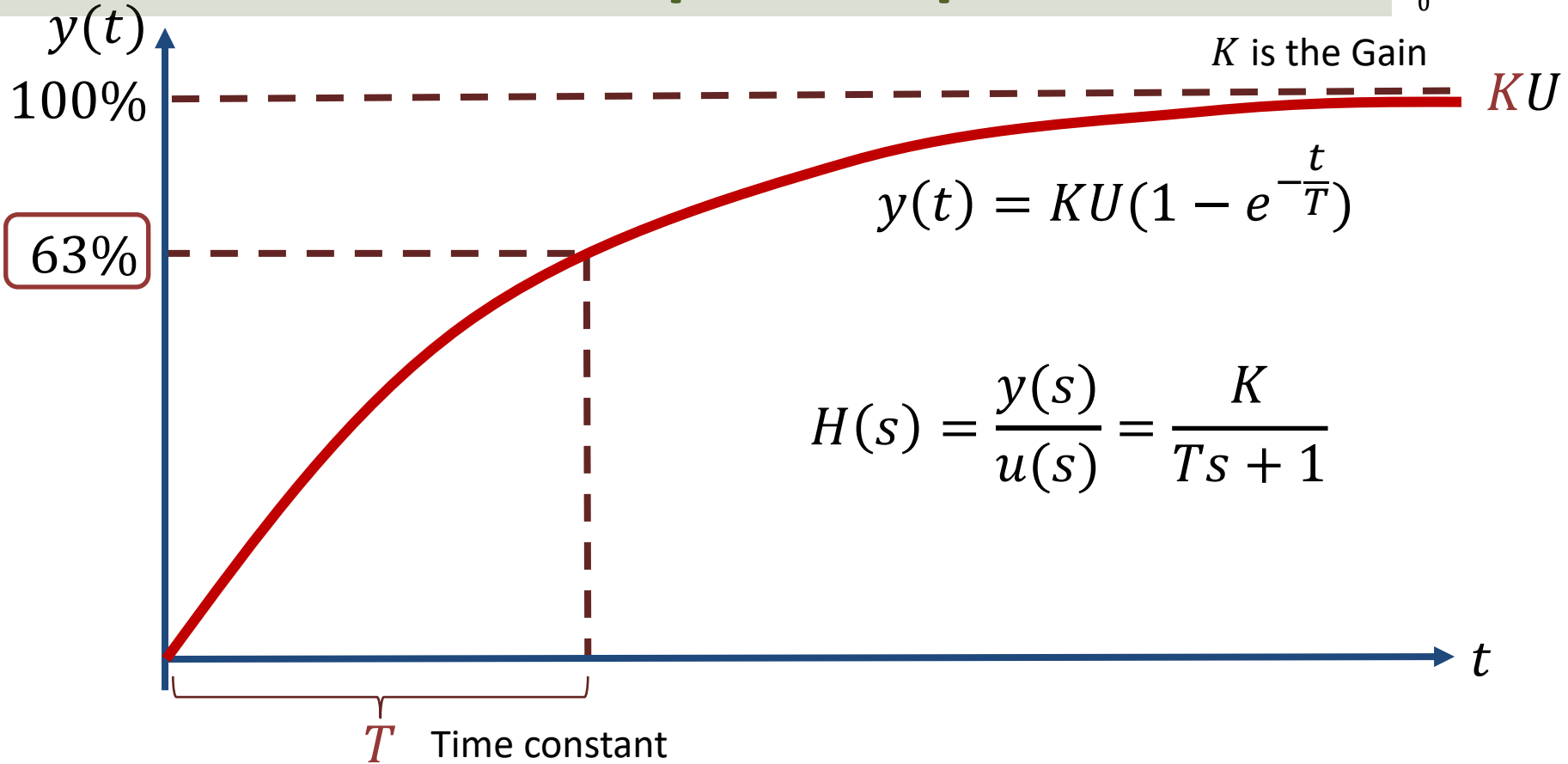
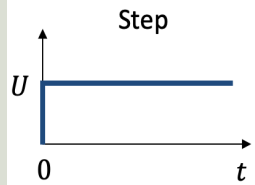
This differential equation represents a 1. order dynamic system

Assume $u(t)$ is a step (U), then we can find that the **solution** to the differential equation is:

$$y(t) = KU(1 - e^{-\frac{t}{T}})$$

(by using Laplace)

1. Order Step Response



Find Solution for Diff. Equation

Given the 1. order System:

$$\dot{y} = \frac{1}{T}(-y + Ku)$$

Let's use Laplace:

$$sy(s) = -\frac{1}{T}y(s) + \frac{K}{T}u(s)$$

$$y(s) \left[s + \frac{1}{T} \right] = \frac{K}{T}u(s)$$

$$y(s) = \frac{K}{Ts + 1}u(s)$$

Then let's assume a Unit Step and use the Laplace Transformation pair $u(t) \Leftrightarrow \frac{U}{s}$

$$y(s) = \frac{K}{Ts + 1} \cdot \frac{U}{s} = \frac{K}{(Ts + 1)s} \cdot U$$

Then we use the Laplace Transformation pair $\frac{K}{(Ts+1)s} \Leftrightarrow K(1 - e^{-\frac{t}{T}})$ to transform the system back to the time domain. This gives the following solution:

$$y(t) = KU(1 - e^{-\frac{t}{T}})$$

The Laplace Transformation pairs can be found in a Table of Laplace Transformations, e.g.,

<https://www.intmath.com/laplace-transformation/table-laplace-transforms.php>

Simulation

The 1. order System:

$$\dot{y} = \frac{1}{T}(-y + Ku)$$

Has the following known solution:

$$y(t) = KU(1 - e^{-\frac{t}{T}})$$

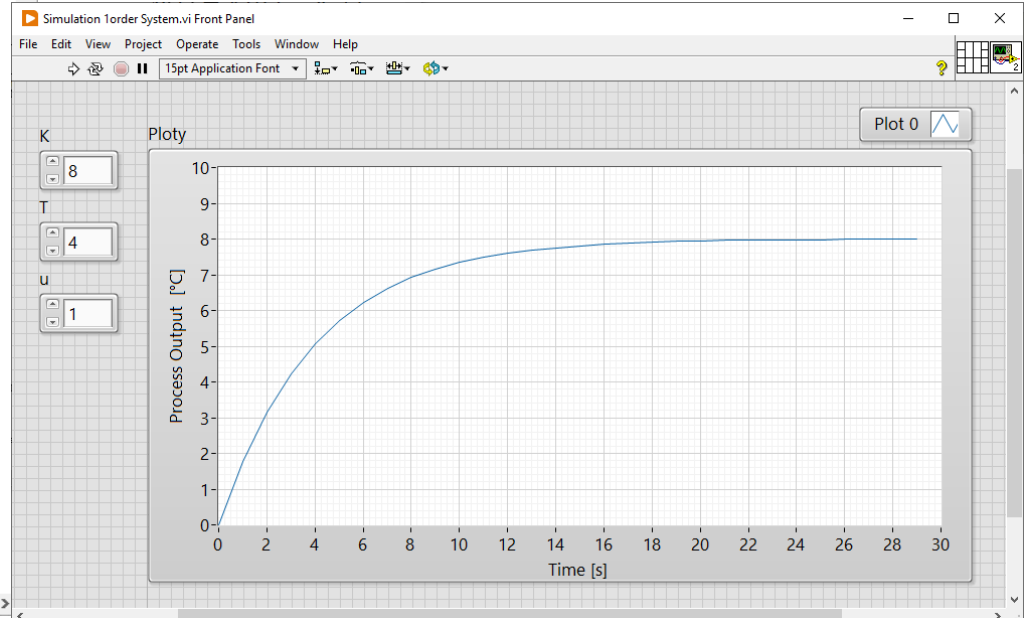
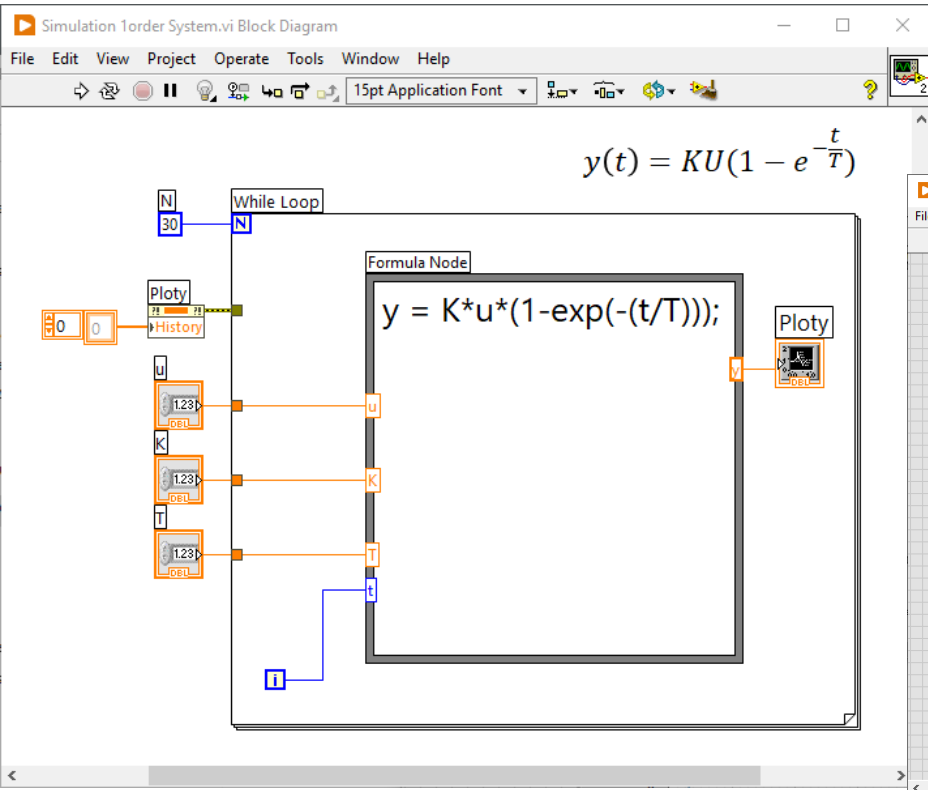
(You can use Laplace to find the solution, see previous page)

Let's Simulate this system in LabVIEW

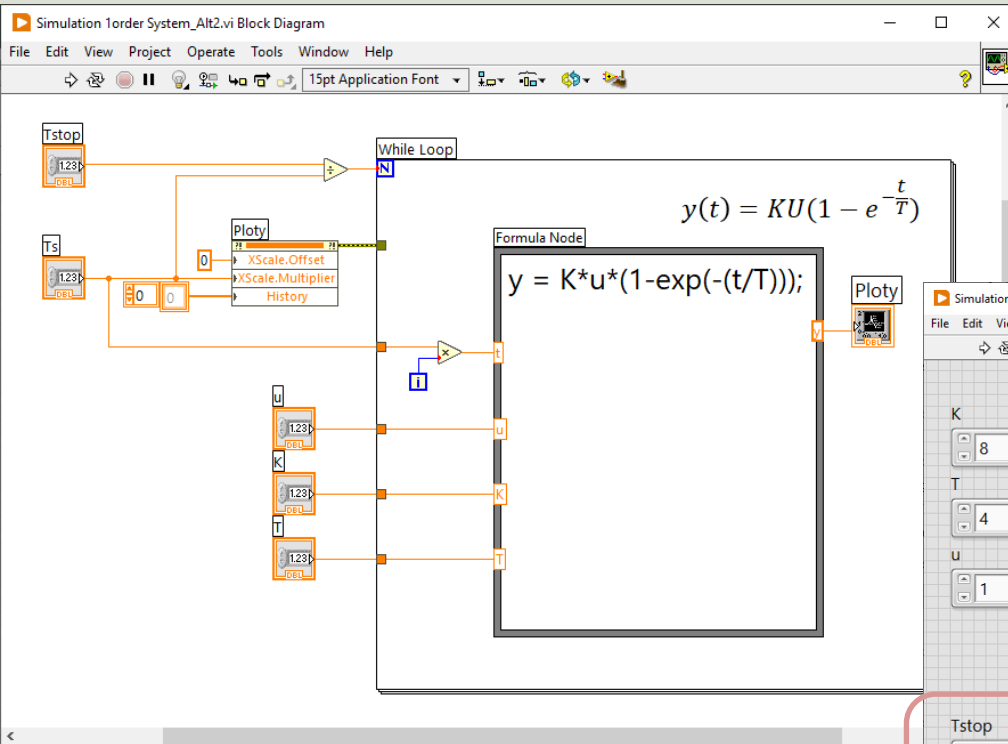
Simulation - LabVIEW

$y(t) = KU(1 - e^{-\frac{t}{T}})$ Let's start with $K = 8$ and $T = 4$ and a step $U = 1$

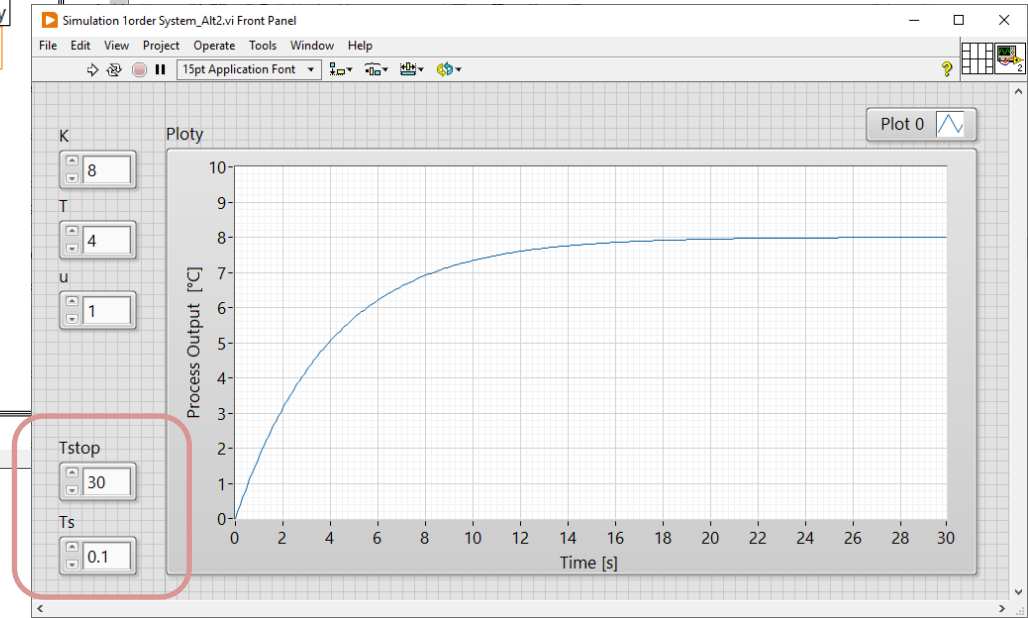
Here we have implemented the equation inside a **Formula Node**. In that way we can write the equations as we do on paper.



Improved Version



Here we can set the Sampling Time T_s and the simulation time T_{stop} from the GUI



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Discretization

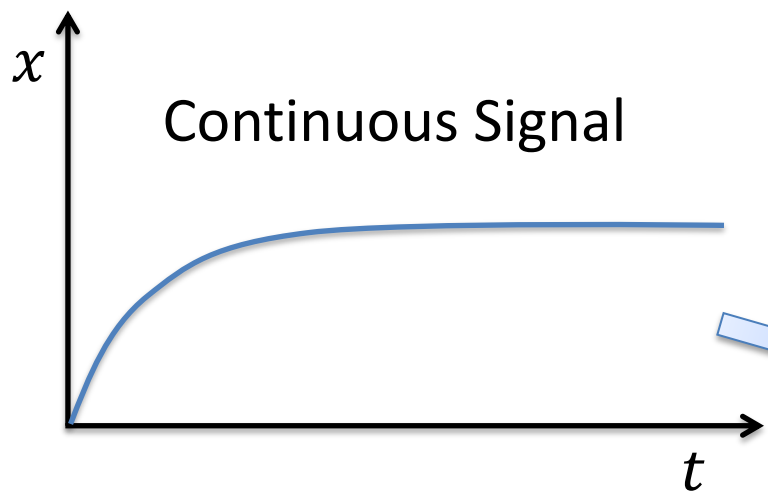


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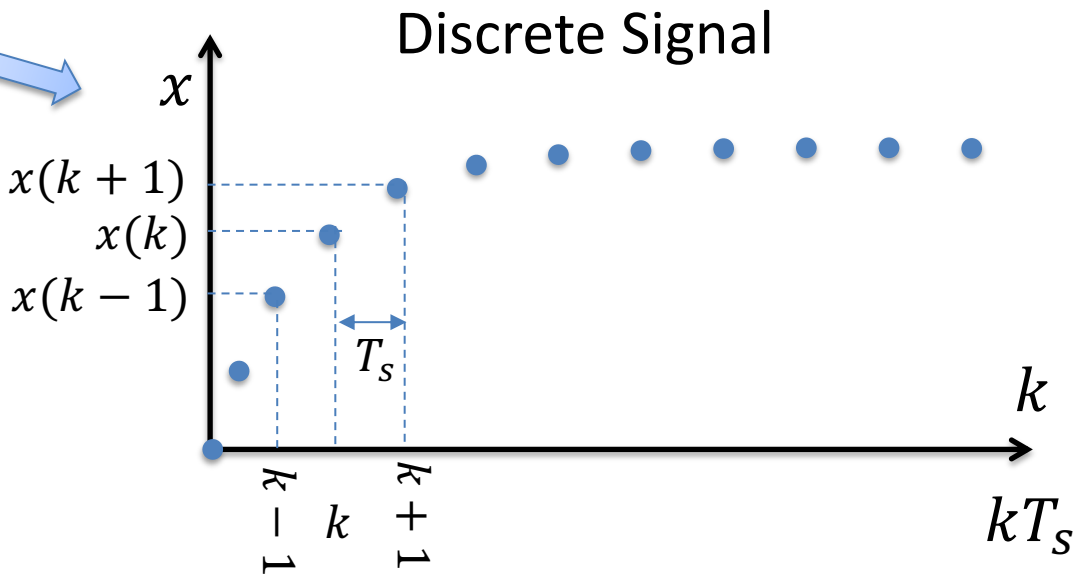
Continuous vs. Discrete Systems

A computer can only deal with discrete signals



Continuous Signal

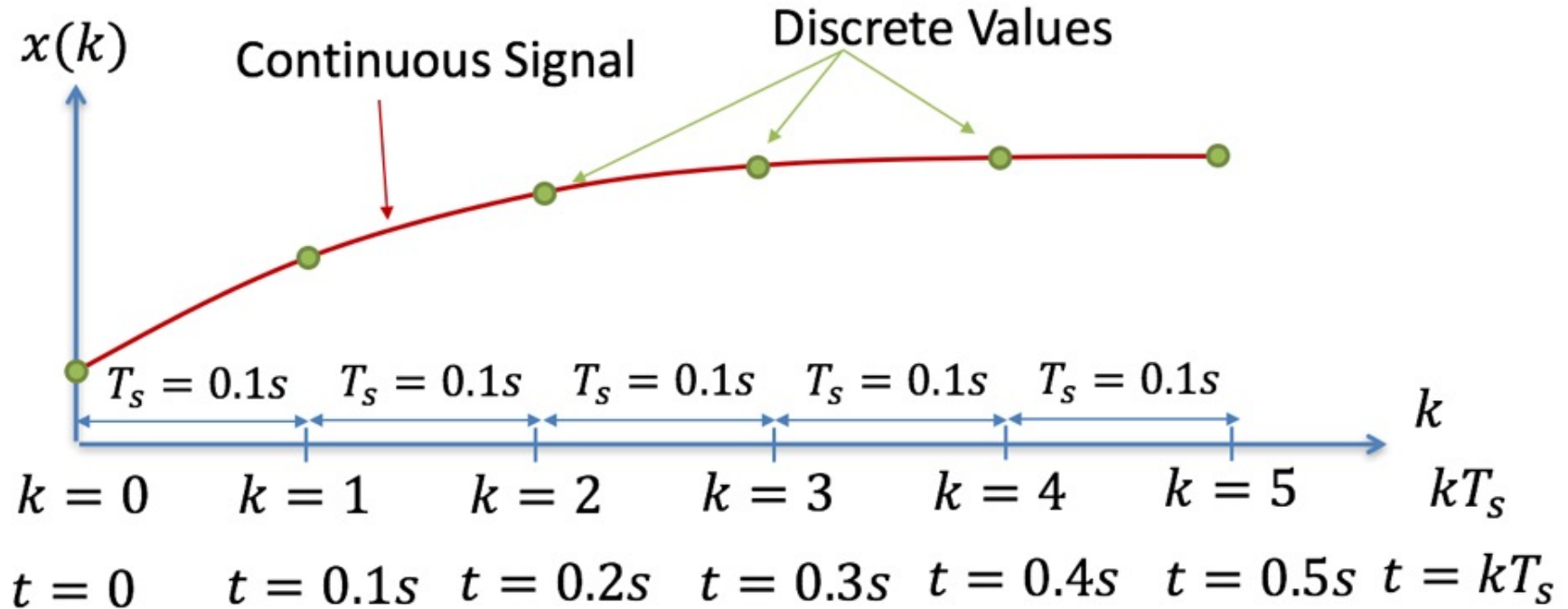
- T_s - Sampling Interval
- $x(k - 1)$ - Previous Value
- $x(k)$ - Current Value
- $x(k + 1)$ - Next Value



Discrete Signal

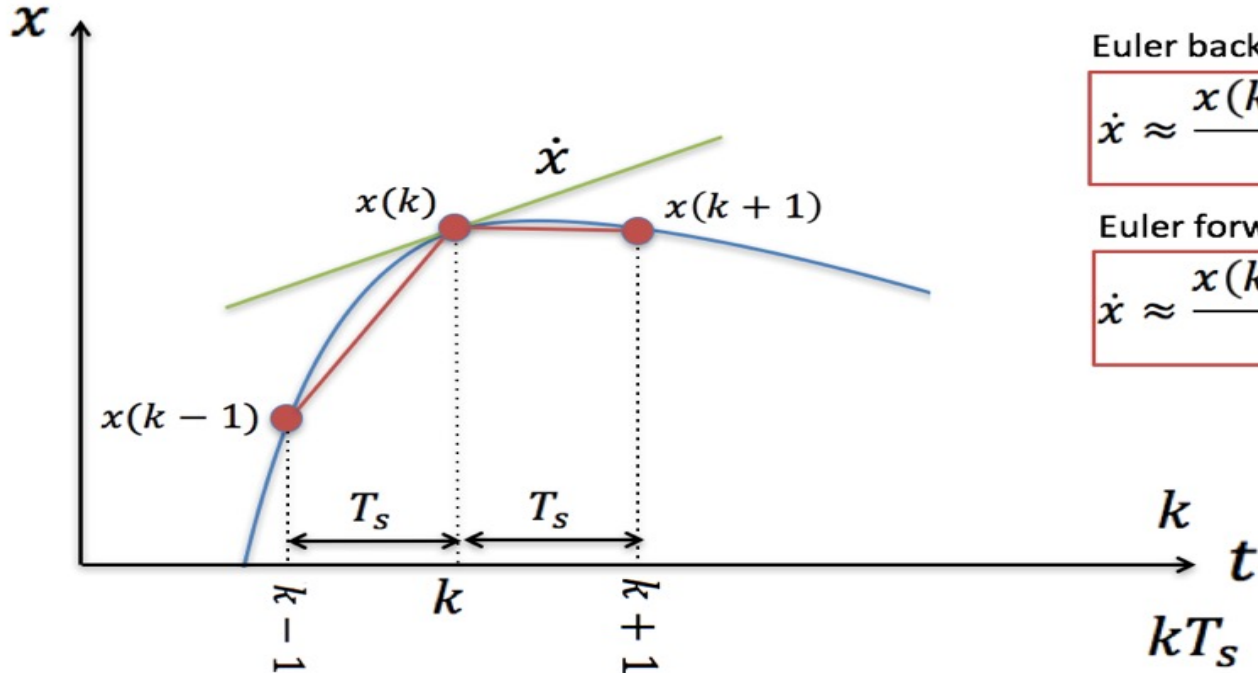
Discrete Data

Below we see a continuous signal vs the discrete signal for a given system with discrete time interval $T_s = 0.1s$. For continuous systems we use t , while for discrete intervals we use k .



Euler Forward method

A simple discretization method is the Euler Forward method



Euler backward method:

$$\dot{x} \approx \frac{x(k) - x(k-1)}{T_s}$$

Euler forward method:

$$\dot{x} \approx \frac{x(k+1) - x(k)}{T_s}$$

Lots of other discretization methods do exist, such as Euler backward, Zero Order Hold (ZOH), Tustin's method, etc.

Discretization

We have a general continuous differential equation:

$$\frac{dx}{dt} = f(t)$$

We can use Euler:

$$\frac{dx}{dt} \approx \frac{x(k+1) - x(k)}{T_s}$$

Then we get:

$$\frac{x(k+1) - x(k)}{T_s} = f(k)$$

This gives the following discrete differential equation:

$$x(k+1) = x(k) + T_s f(k)$$

Discretization

We have the continuous differential equation: $\dot{x} = -ax + bu$

We apply **Euler**: $\dot{x} \approx \frac{x(k+1) - x(k)}{T_s}$

Then we get:

$$\frac{x(k+1) - x(k)}{T_s} = -ax(k) + bu(k)$$

This gives the following discrete differential equation (difference equation):

$$x(k+1) = (1 - T_s a)x(k) + T_s bu(k)$$

This equation can easily be implemented in any text-based programming language or the Formula Node in LabVIEW

Where $a = \frac{1}{T}$ and $b = \frac{K}{T}$

Discrete Model in LabVIEW

$$x(k + 1) = (1 - T_s a)x(k) + T_s b u(k)$$

Discrete Model.vi Front Panel

U: 1

Process Value [°C]: 0.00

Model Parameters

- K: 1
- T: 4
- Initial Value [°C]: 0.00
- Ts: 0.1

Discrete Model.vi Block Diagram

Formula Node

```
float a = 1/T;
float b = K/T;
xk1 = (1-Ts*a)*xk + Ts*b*uk;
```

Model Parameters

- K
- T
- U
- Ts

Feedback Node

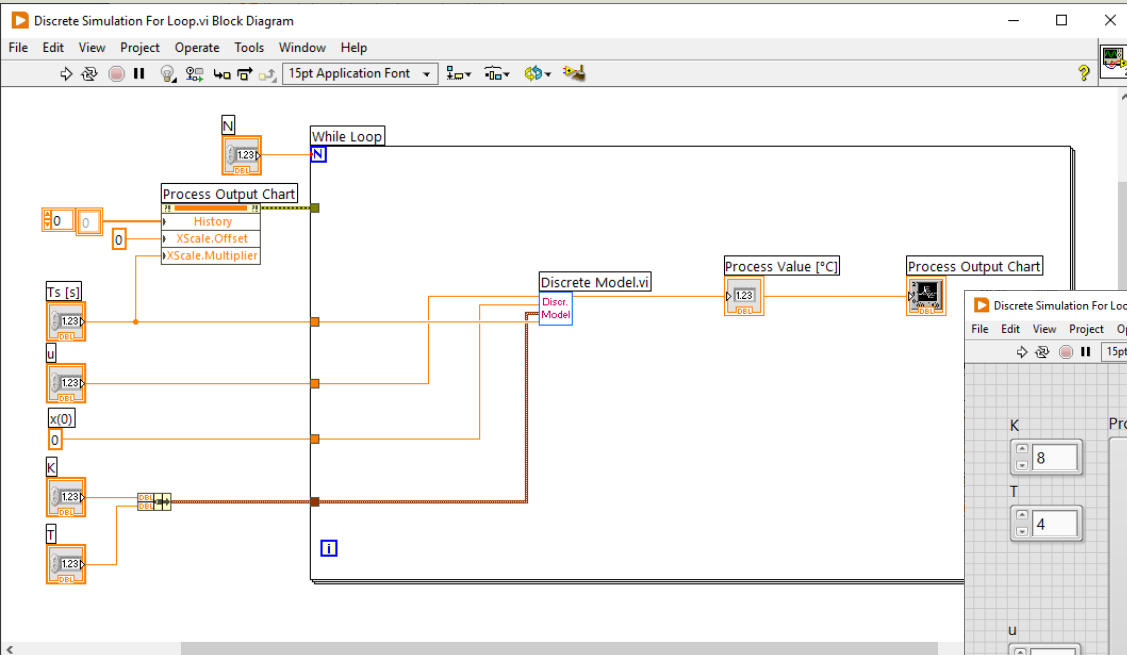
Initial Value [°C]

Process Value [°C]

Here we have implemented the model inside a **Formula Node**. In that way we can write the equations as we do on paper.

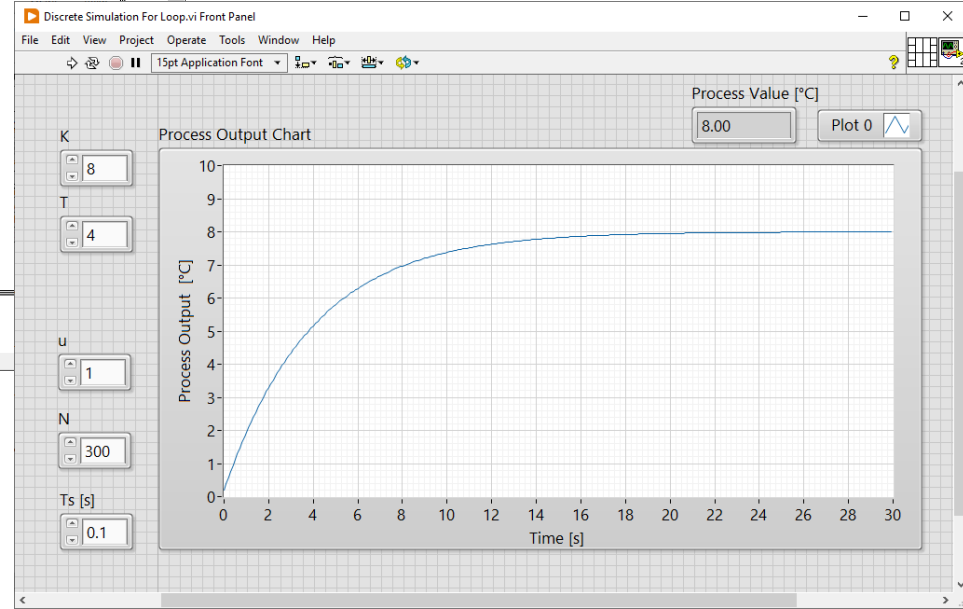
We have also implemented the model as a **SubVI** then we can easily reuse the model in different applications

Simulation Example



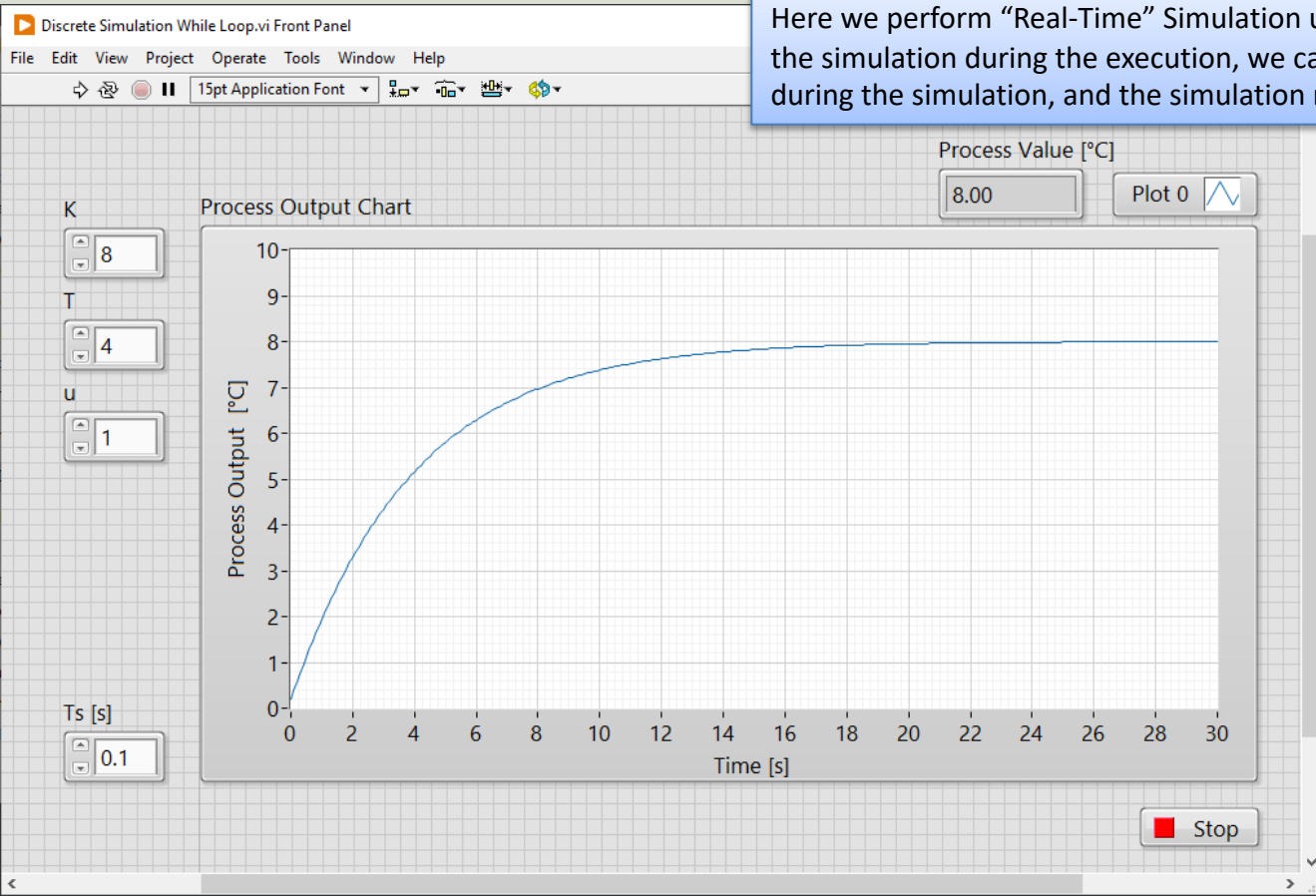
Here we have used a standard **For Loop** in our simulation. We specify the length of the Simulation with **N** (Number of Iterations)

$$T_{stop} = N \times T_s = 300 \times 0.1s = 30s$$

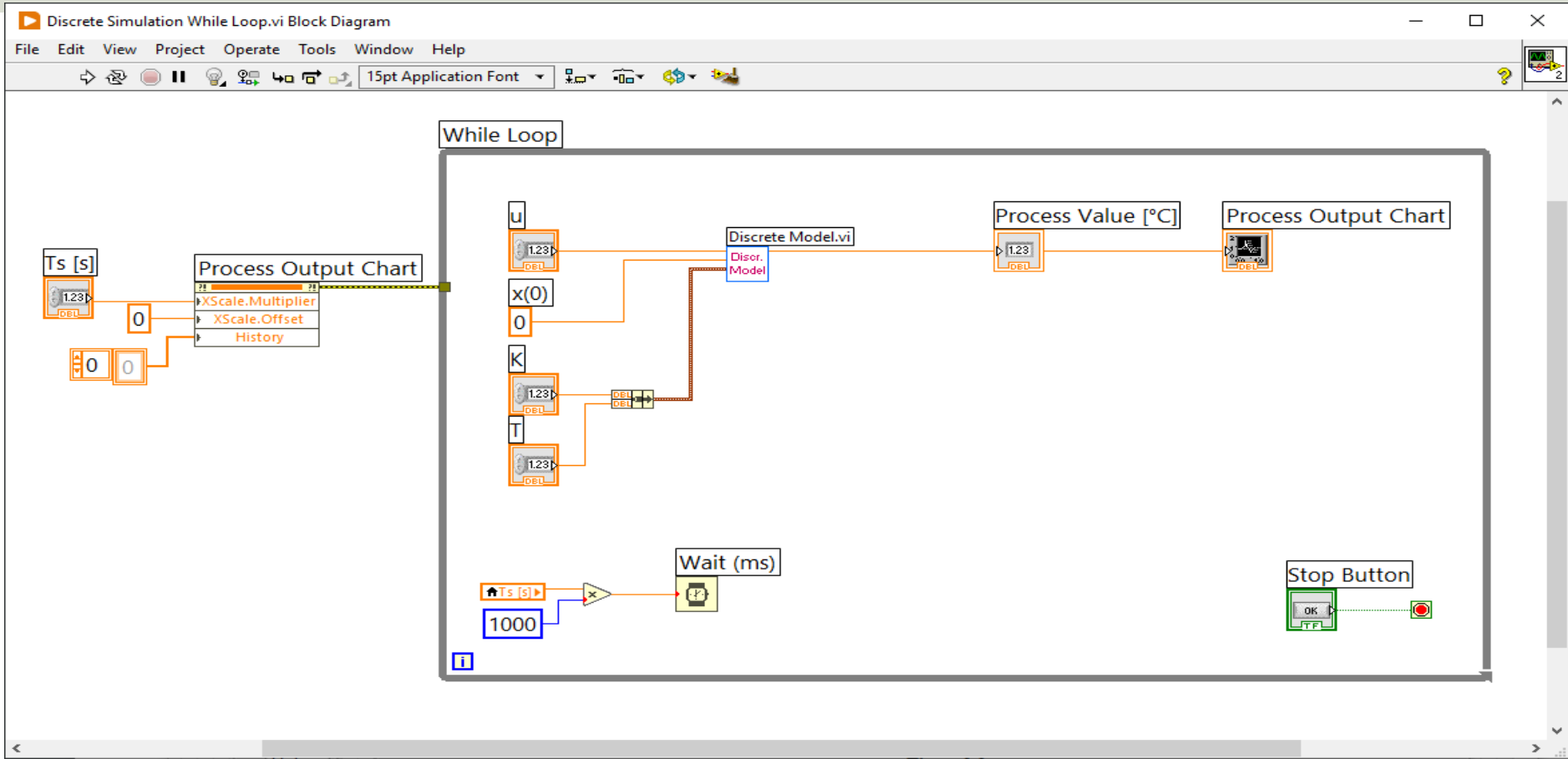


“Real-Time” Simulation in LabVIEW

Here we perform “Real-Time” Simulation using a **While Loop**. Here we can interact with the simulation during the execution, we can change simulation parameters any time during the simulation, and the simulation runs forever until we click the Stop button.



LabVIEW Code



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Simulation Examples

2.order System

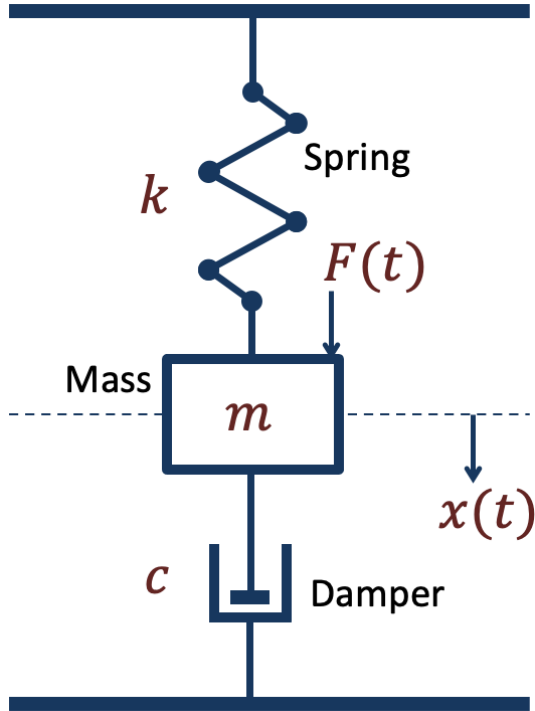
Mass-Spring-Damper System

Hans-Petter Halvorsen



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Mass-Spring-Damper System



The “Mass-Spring-Damper” System is typical system used to demonstrate and illustrate Modelling and Simulation Applications

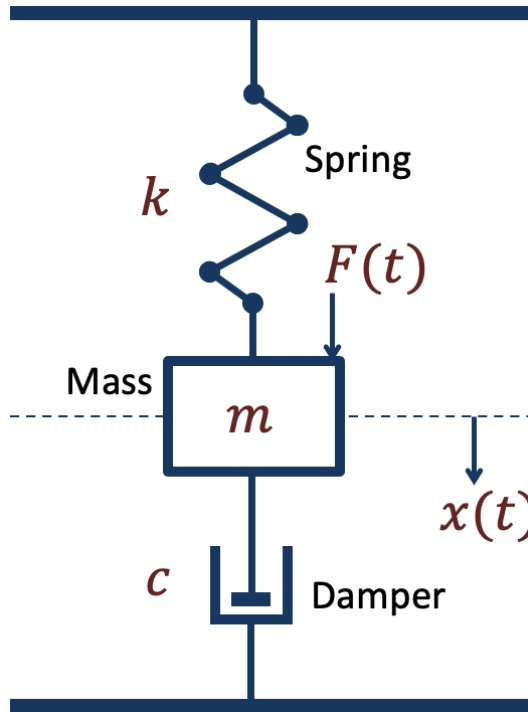
$$F(t) - c\dot{x}(t) - kx(t) = m\ddot{x}(t)$$

Force - F [N], Spring constant - k [N/m], Damping coefficient - c [kg/s], Position - x [m/s],
Velocity - \dot{x} [m/s²], Acceleration - \ddot{x} [m/s²], Mass - m [kg]

Mass-Spring-Damper System

Given a so-called "Mass-Spring-Damper" system

Newtons 2.law: $\sum F = ma$



The system can be described by the following equation:

$$F(t) - c\dot{x}(t) - kx(t) = m\ddot{x}(t)$$

Where t is the time, $F(t)$ is an external force applied to the system, c is the damping constant, k is the stiffness of the spring, m is a mass.

$x(t)$ is the position of the object (m)

$\dot{x}(t)$ is the first derivative of the position, which equals the velocity/speed of the object (m)

$\ddot{x}(t)$ is the second derivative of the position, which equals the acceleration of the object (m)

Mass-Spring-Damper System

$$F(t) - c\dot{x}(t) - kx(t) = m\ddot{x}(t)$$

$$m\ddot{x} = F - c\dot{x} - kx$$

$$\ddot{x} = \frac{1}{m}(F - c\dot{x} - kx)$$

We set

$$x = x_1$$

$$\dot{x} = x_2$$

Finally:

$$\ddot{x} = \frac{1}{m}(F - c\dot{x} - kx)$$



$$\begin{aligned}\dot{x}_1 &= x_2 \\ \dot{x}_2 &= \frac{1}{m}(F - cx_2 - kx_1)\end{aligned}$$

Higher order differential equations can typically be reformulated into a system of first order differential equations

x_1 = Position

x_2 = Velocity/Speed

Discretization

Given:

$$\begin{aligned}\dot{x}_1 &= x_2 \\ \dot{x}_2 &= \frac{1}{m}(F - cx_2 - kx_1)\end{aligned}$$

Using Euler:

$$\dot{x} \approx \frac{x(k+1) - x(k)}{T_s}$$

Then we get:

$$\begin{aligned}\frac{x_1(k+1) - x_1(k)}{T_s} &= x_2(k) \\ \frac{x_2(k+1) - x_2(k)}{T_s} &= \frac{1}{m}[F(k) - cx_2(k) - kx_1(k)]\end{aligned}$$

This gives:

$$\begin{aligned}x_1(k+1) &= x_1(k) + T_s x_2(k) \\ x_2(k+1) &= x_2(k) + T_s \frac{1}{m}[F(k) - cx_2(k) - kx_1(k)]\end{aligned}$$

Then we get:

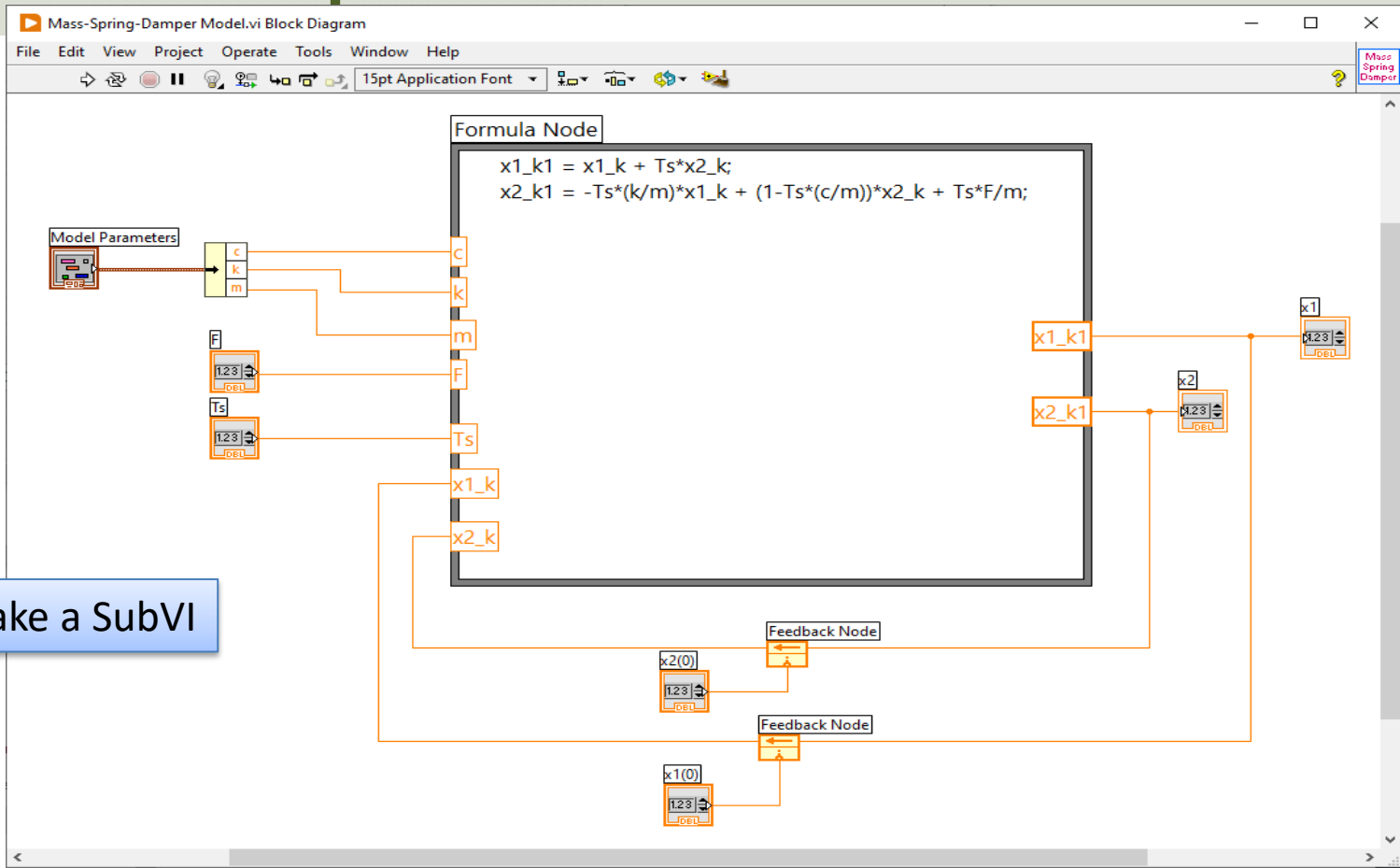
$$\begin{aligned}x_1(k+1) &= x_1(k) + T_s x_2(k) \\ x_2(k+1) &= -T_s \frac{k}{m} x_1(k) + x_2(k) - T_s \frac{c}{m} x_2(k) + T_s \frac{1}{m} F(k)\end{aligned}$$

Finally:

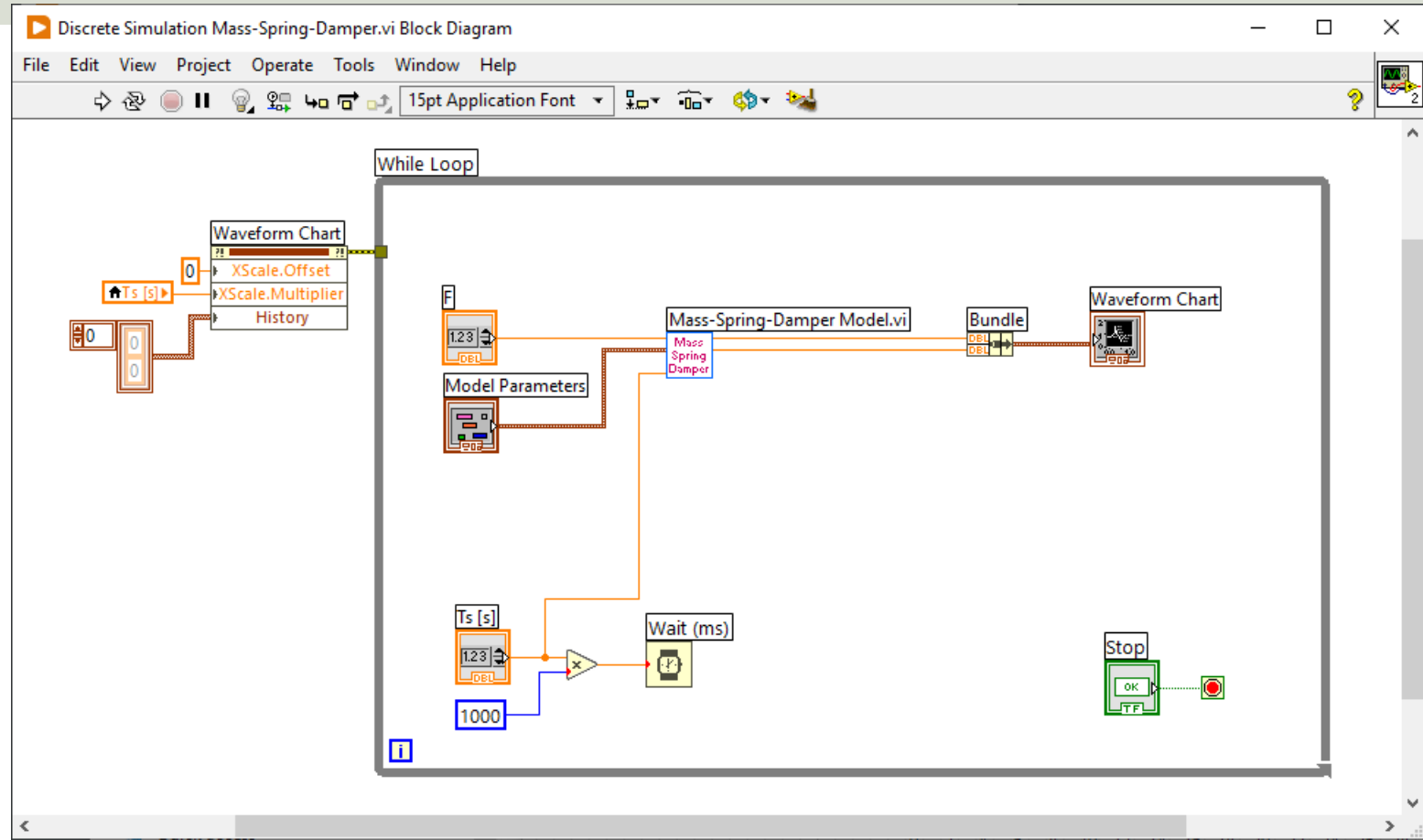
$$\begin{aligned}x_1(k+1) &= x_1(k) + T_s x_2(k) \\ x_2(k+1) &= -T_s \frac{k}{m} x_1(k) + (1 - T_s \frac{c}{m}) x_2(k) + T_s \frac{1}{m} F(k)\end{aligned}$$

This can be implemented in LabVIEW

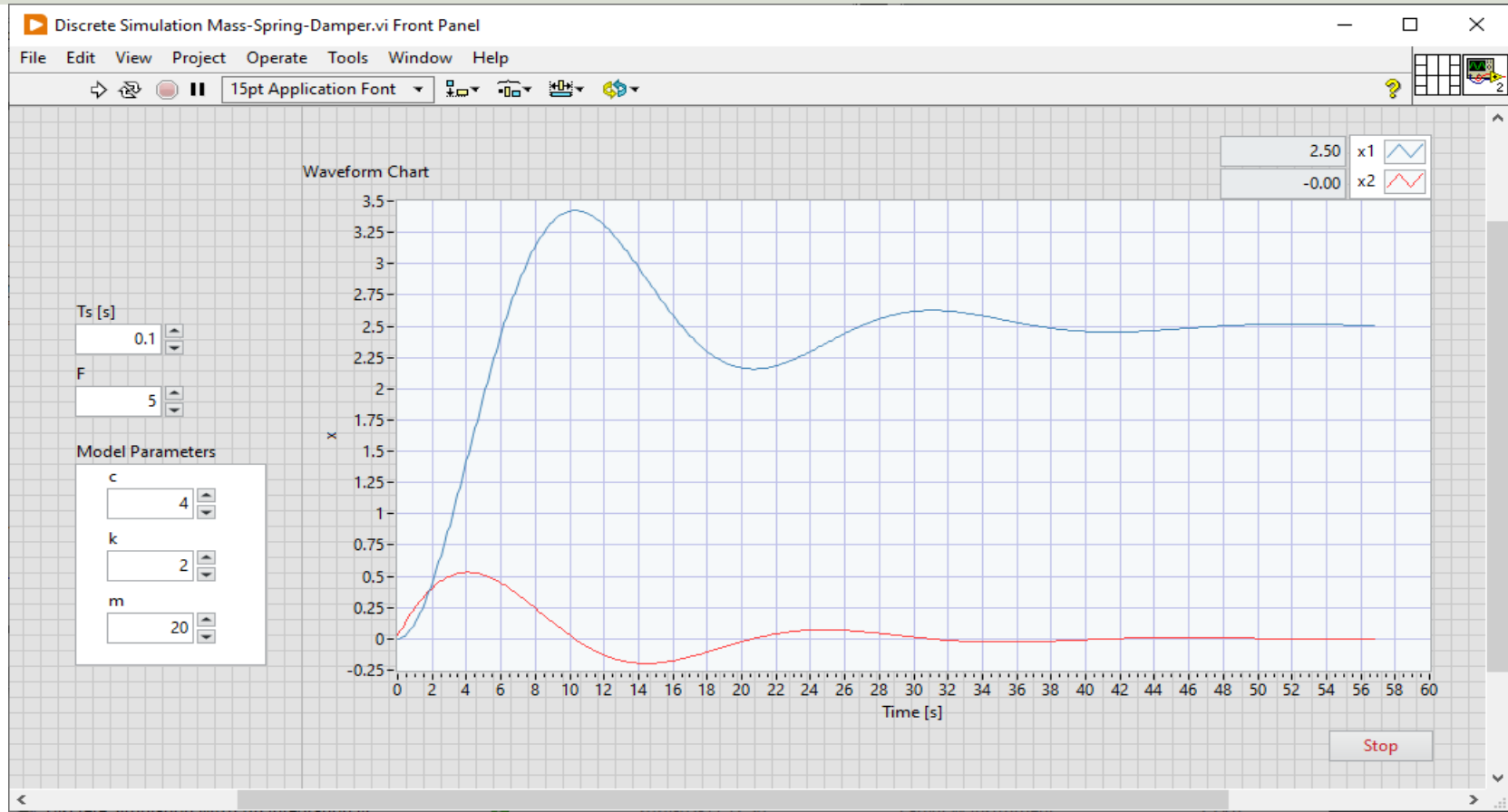
Model Implementation in LabVIEW



Simulation in LabVIEW



LabVIEW



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ODE Functions in LabVIEW















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ODE Functions in LabVIEW



Mathematics

↑ Search Customize

 Numeric	 Elementary	 Linear Algebra
 Fitting	 Interp & Extrapolation	 Integ & Diff
 Prob & Stat	 Optimization	 Differential Eqs
 Geometry	 Polynomial	 Script & Formula










Differential Equations

↑ Search Customize

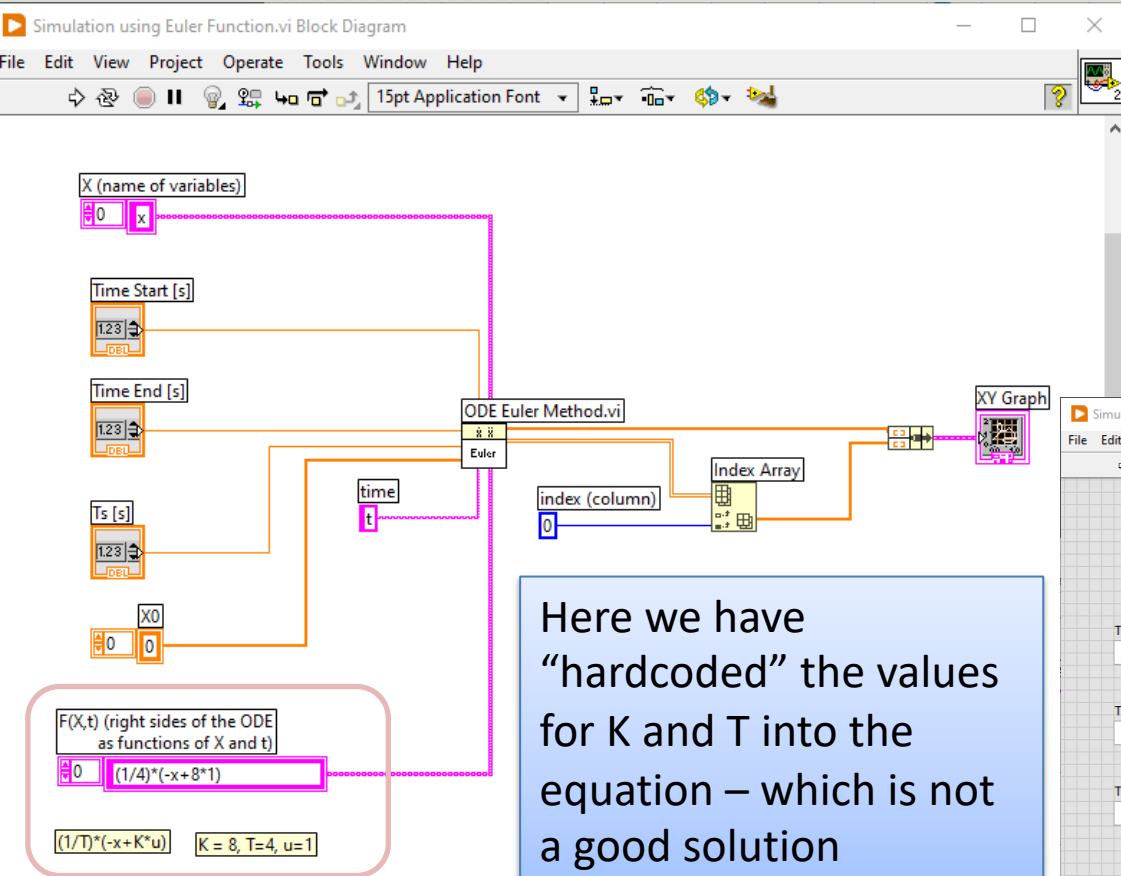
 Ordinary Differential ...	 Partial Differential ...
--	---

Ordinary Differential Equations

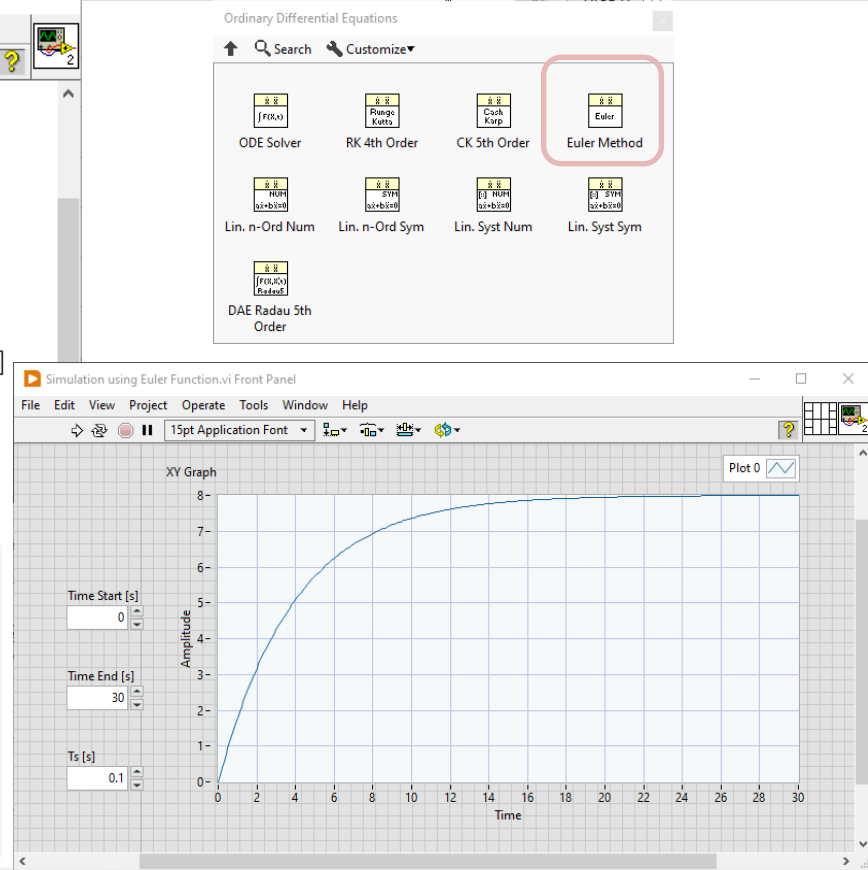
↑ Search Customize

 ODE Solver	 RK 4th Order	 Cash Karp	 Euler Method
 Lin. n-Ord Num	 Lin. n-Ord Sym	 Lin. Syst Num	 Lin. Syst Sym
 DAE Radau 5th Order			

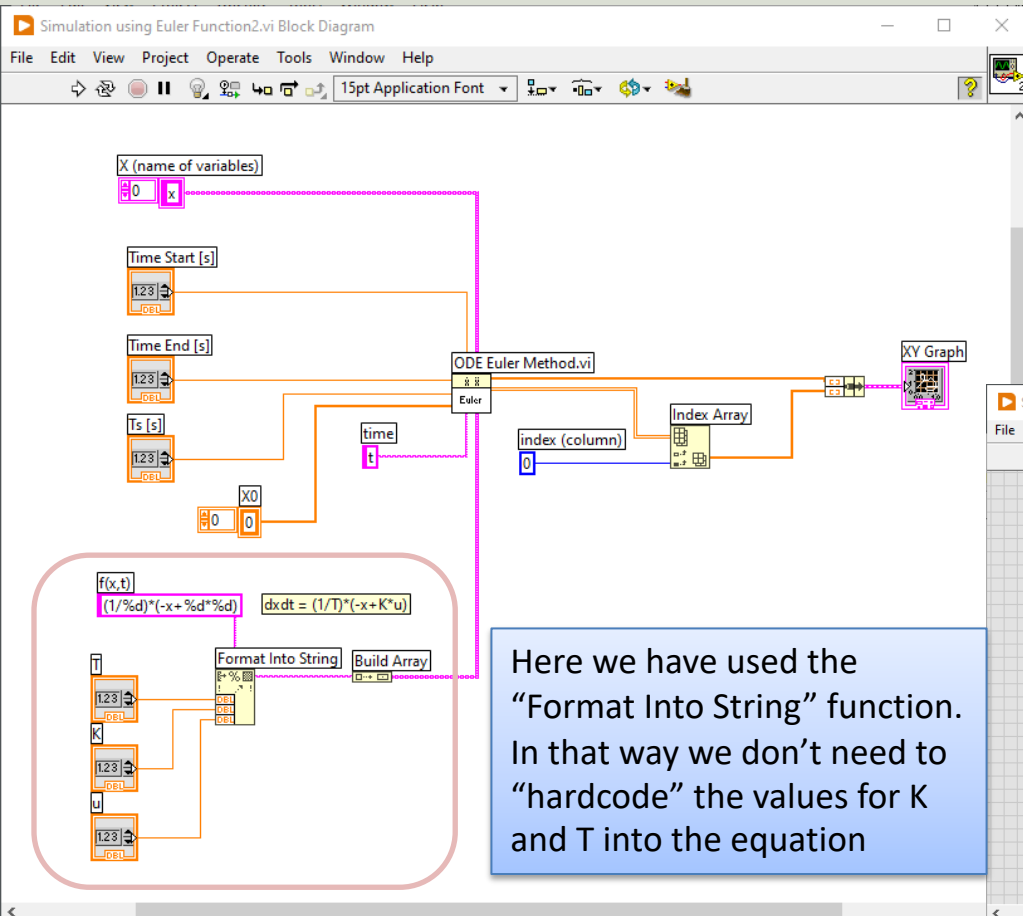
ODE Euler Method.vi



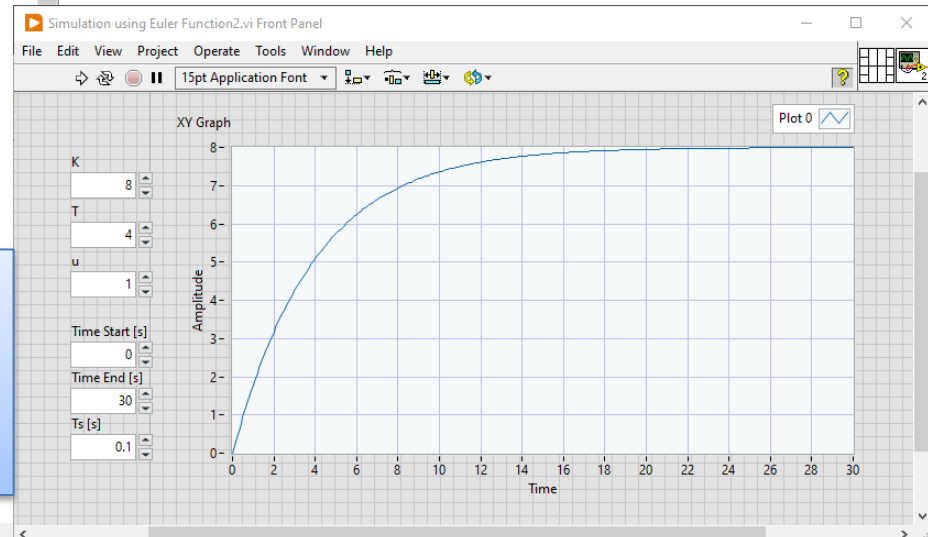
Here we have “hardcoded” the values for K and T into the equation – which is not a good solution



ODE Euler Method.vi – Alt2

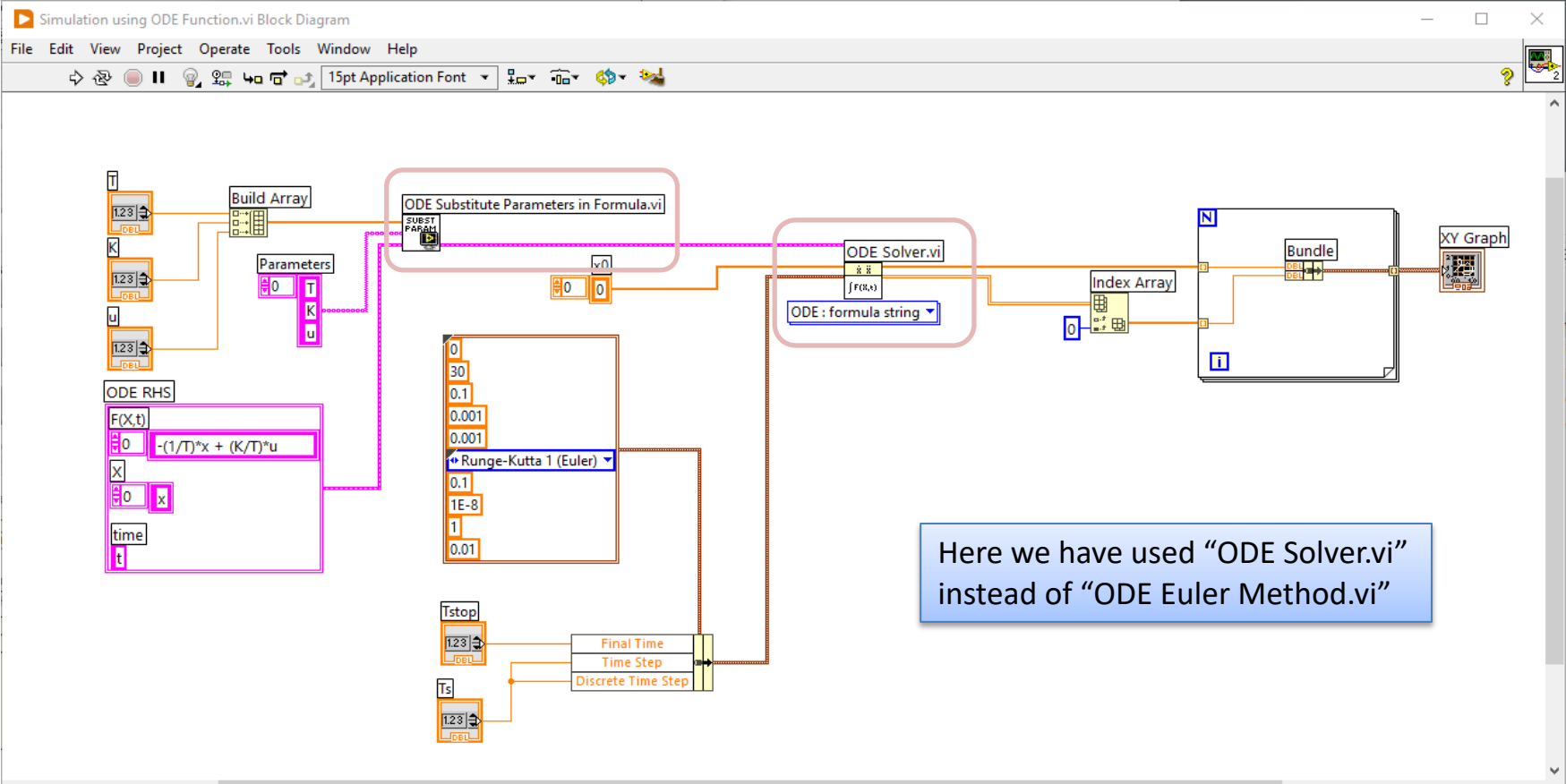


Here we have used the “Format Into String” function. In that way we don’t need to “hardcode” the values for K and T into the equation



ODE Solver.vi

ODE Solver.vi
Automatic
ODE Solver
F(X,t) is VI
F(x,t) is formula string

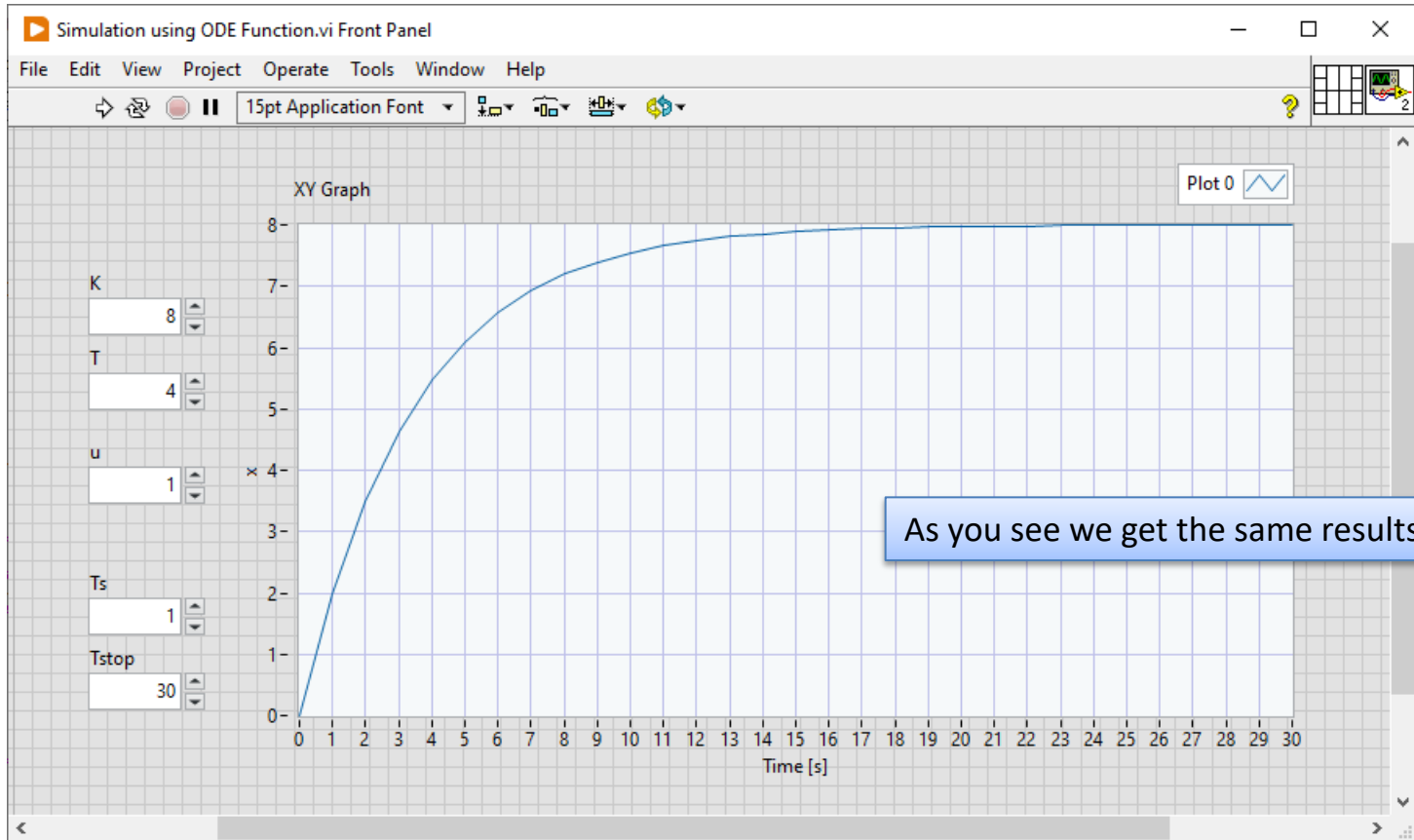


Here we have used "ODE Solver.vi" instead of "ODE Euler Method.vi"

ODE Solver.vi

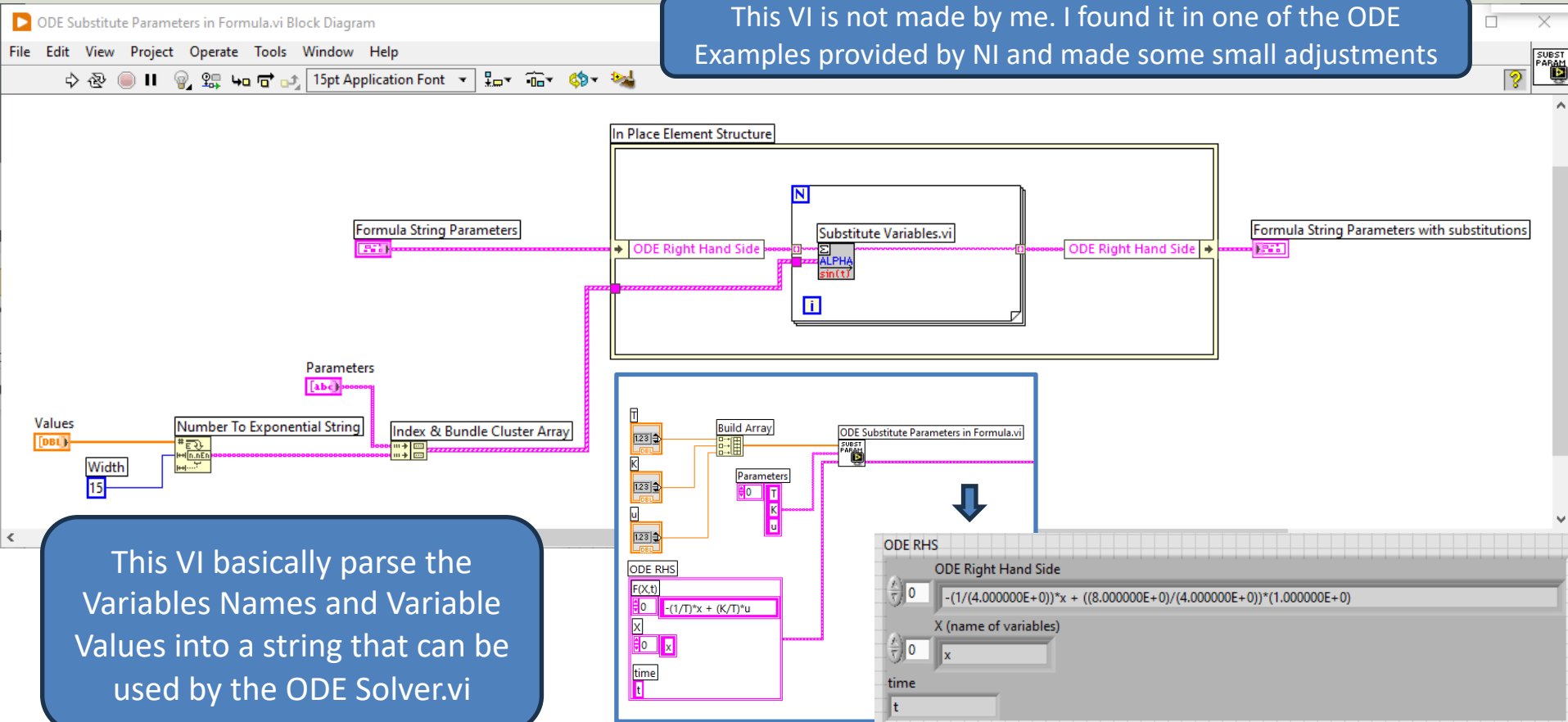
ODE Solver.vi
F(x,t)

Automatic
ODE Solver
F(x,t) is VI
F(x,t) is formula string



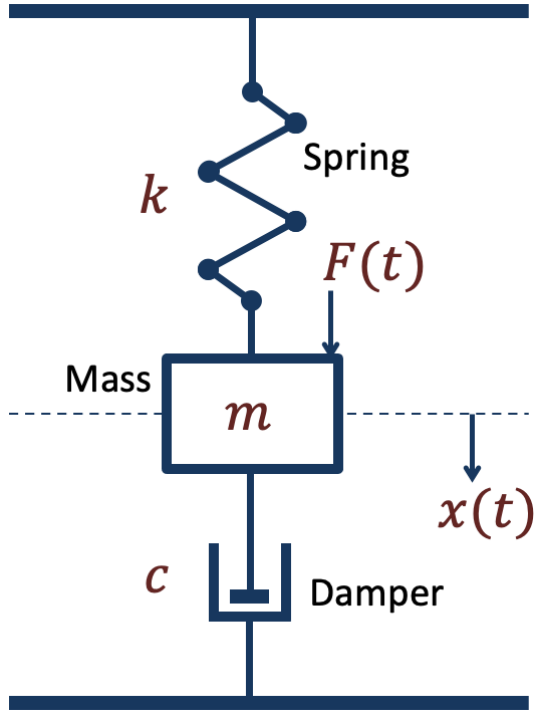
ODE Substitute Parameters in Formula.vi

This VI is not made by me. I found it in one of the ODE Examples provided by NI and made some small adjustments



This VI basically parse the Variables Names and Variable Values into a string that can be used by the ODE Solver.vi

Mass-Spring Damper System



See if you can use “ODE Solver.vi” for implementing and simulating the Mass-Spring Damper System as well (I leave that to you)

$$\begin{aligned}\dot{x}_1 &= x_2 \\ \dot{x}_2 &= \frac{1}{m}(F - cx_2 - kx_1)\end{aligned}$$

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Python Integration



Hans-Petter Halvorsen

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Python Integration

Functions Programming

- Structures
- Array
- Cluster, Class, & Variant
- Numeric
- Boolean
- String
- Comparison
- Waveform
- Collection
- File I/O
- Timing
- Dialog & User Interface
- Synchronization
- Graphics & Sound
- Application Control
- Report Generation
- VI Analyzer
- Desktop Execution Trac...
- Unit Test Framework

- Measurement I/O
- Instrument I/O
- Mathematics
- Signal Processing
- Data Communication
- Connectivity**
- Control & Simulation
- Express
- Addons
- Select a VI...
- DSC Module
- MakerHub

Connectivity

- Libraries & Executables
- .NET
- Python
- Input Device Control
- MATLAB(R)
- ActiveX
- Database

Python

↑ Search Customize

- Open Python Session
- Open Virtual Env Session.vi
- Python Object Refnum ...
- Python Node
- Close Python Session
- Close Reference

Python Integration Example

```
def c2f(Tc) :  
    Tf = (Tc * 9/5) + 32  
    return Tf  
  
def f2c(Tf) :  
    Tc = (Tf - 32) * (5/9)  
    return Tc
```

fahrenheit.py

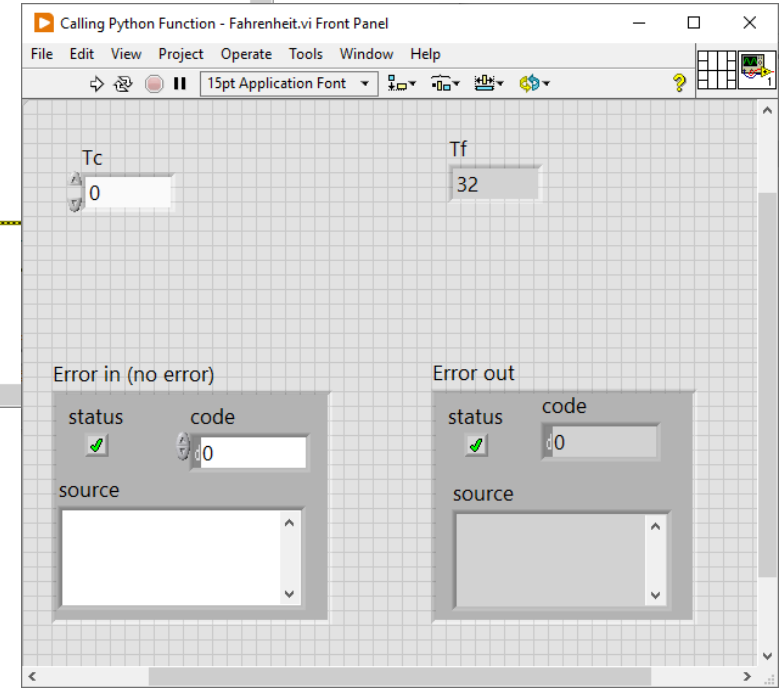
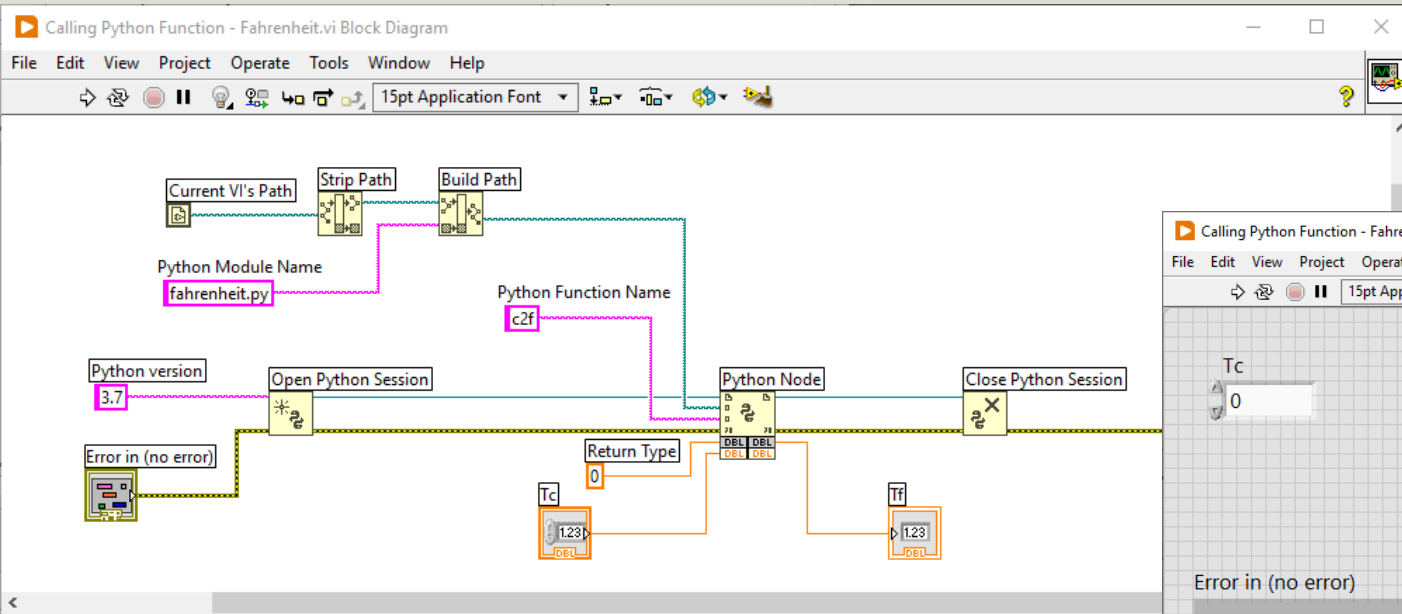
We make a Python Module with 2 Functions, one that converts from Celsius to Fahrenheit and another that converts from Fahrenheit to Celsius

We start by making the Python code using Spyder or another Python Editor

We test if it works:

```
from fahrenheit import c2f, f2c  
  
Tc = 0  
  
Tf = c2f(Tc)  
  
print("Fahrenheit: " + str(Tf))  
  
Tf = 32  
  
Tc = f2c(Tf)  
  
print("Celsius: " + str(Tc))
```

Python Integration Example



Note! LabVIEW and Python needs to match. If you use a 32bit LabVIEW version, you also need to use a 32bit Python version

Simulation

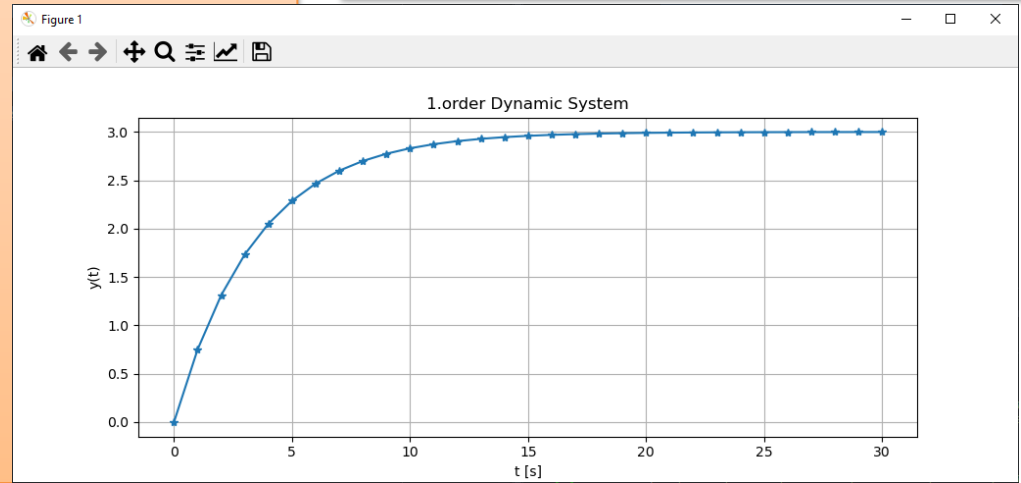
Here we make a discrete simulation example in Python using our 1.order model from previous examples

```
import numpy as np
```

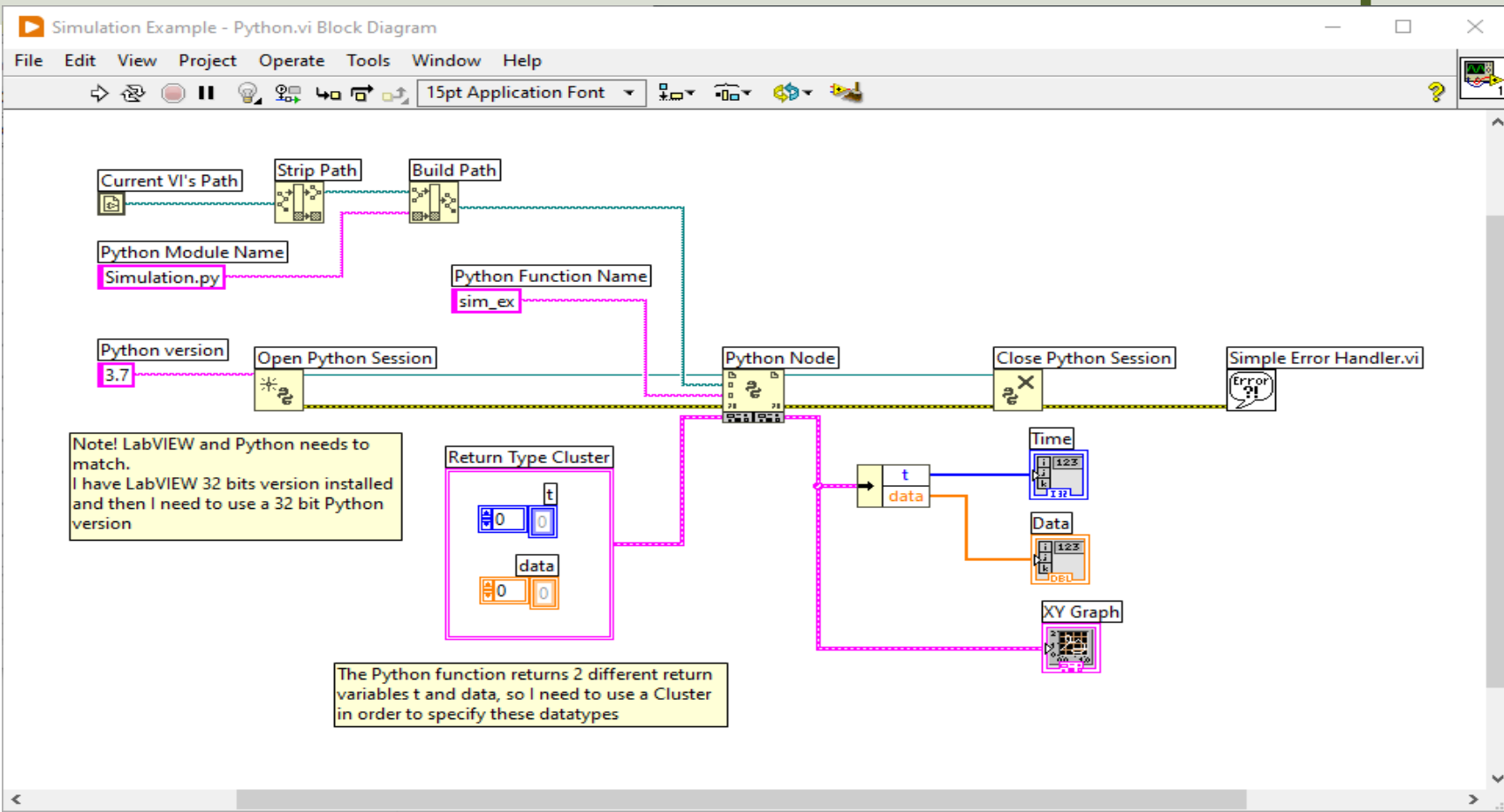
```
def sim_ex():  
    # Model Parameters  
    K = 3  
    T = 4  
    a = -1/T  
    b = K/T  
  
    #Simulation Parameters  
    yk = 0  
    uk = 1  
    Tstop = 30  
    Ts = 1  
    N = int(Tstop/Ts) # Simulation length  
  
    data = []  
    data.append(yk)  
  
    # Simulation  
    for k in range(N):  
        #Model Implementation  
        yk1 = (1 + a*Ts) * yk + Ts*b*uk  
  
        yk = yk1  
        data.append(yk1)  
  
    t = np.arange(0, Tstop+Ts, Ts)  
    return t, data
```

Simulation.py

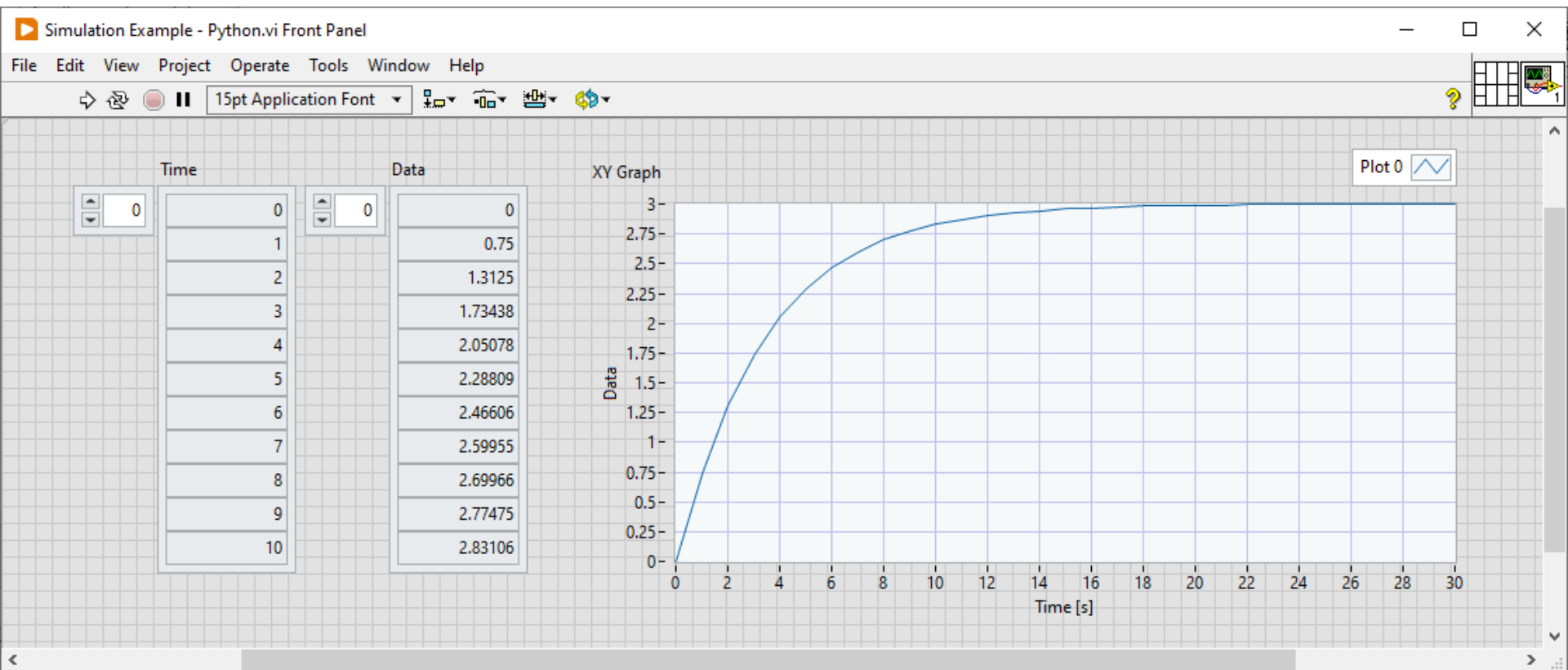
```
import matplotlib.pyplot as plt  
from Simulation import sim_ex  
  
#Run Simulation  
t, data = sim_ex()  
  
# Plot the Simulation Results  
plt.plot(t, data, '-*')  
plt.title('1.order Dynamic System')  
plt.xlabel('t [s]')  
plt.ylabel('y(t)')  
plt.grid()
```



LabVIEW Simulation Example



LabVIEW Simulation Example



Virtual Python Environment

- With Python you can create Virtual Environments.
- Here you can install your independent set of Python packages.
- In that way you can create an isolated environment where you can run your Python Applications/Scripts without destroying for other Applications/Scripts using other versions of different Python packages.
- You can create a Virtual Python Environment using **venv** command:

```
python -m venv /path/to/new/virtual/environment
```
- You can also use tools like Visual Studio, VenviPy, etc. to do this from a user interface.

Virtual Environment with Visual Studio

Solution Explorer

Search Solution Explorer (Ctrl+“)

Solution 'PythonApplication1' (1 of 1 project)

- PythonApplication1
 - Python Environments
 - envhph1 (Python 3.7 (32-bit))
 - numpy (1.21.6)
 - pip (10.0.1)
 - setuptools (39.0.1)
 - References
 - Search Paths

Add Environment...

- View All Python Environments
- Collapse All Descendants Scope to This
- New Solution Explorer View

Python Environments

Add Environment...

- Anaconda 2021.11
- envhph1 (Python 3.7 (32-bit))
 - Overview
 - Packages (PyPI)
 - numpy (1.21.6)
 - Run command: pip install numpy
 - Install numpy-aarch64
 - Install numpy-alignments (0.0.10)
 - Install numpy-allocator (1.2.0)
 - Install numpy-api-bench (0.1.0)
 - Install numpy-array-buffer (0.1.2)
 - Install numpy-camera (0.0.2)
 - Install numpy-choices (0.10)
 - Install numpy-cloud (0.0.5)
 - Install numpy-cursor (1.0.5)
 - Install numpy-dataframe (0.1.7)
 - Install numpy1 (0.0.1)
- Python 3.6 (64-bit)
- Python 3.7 (32-bit)
- Python 3.7 (64-bit)
- Python 3.9 (64-bit)

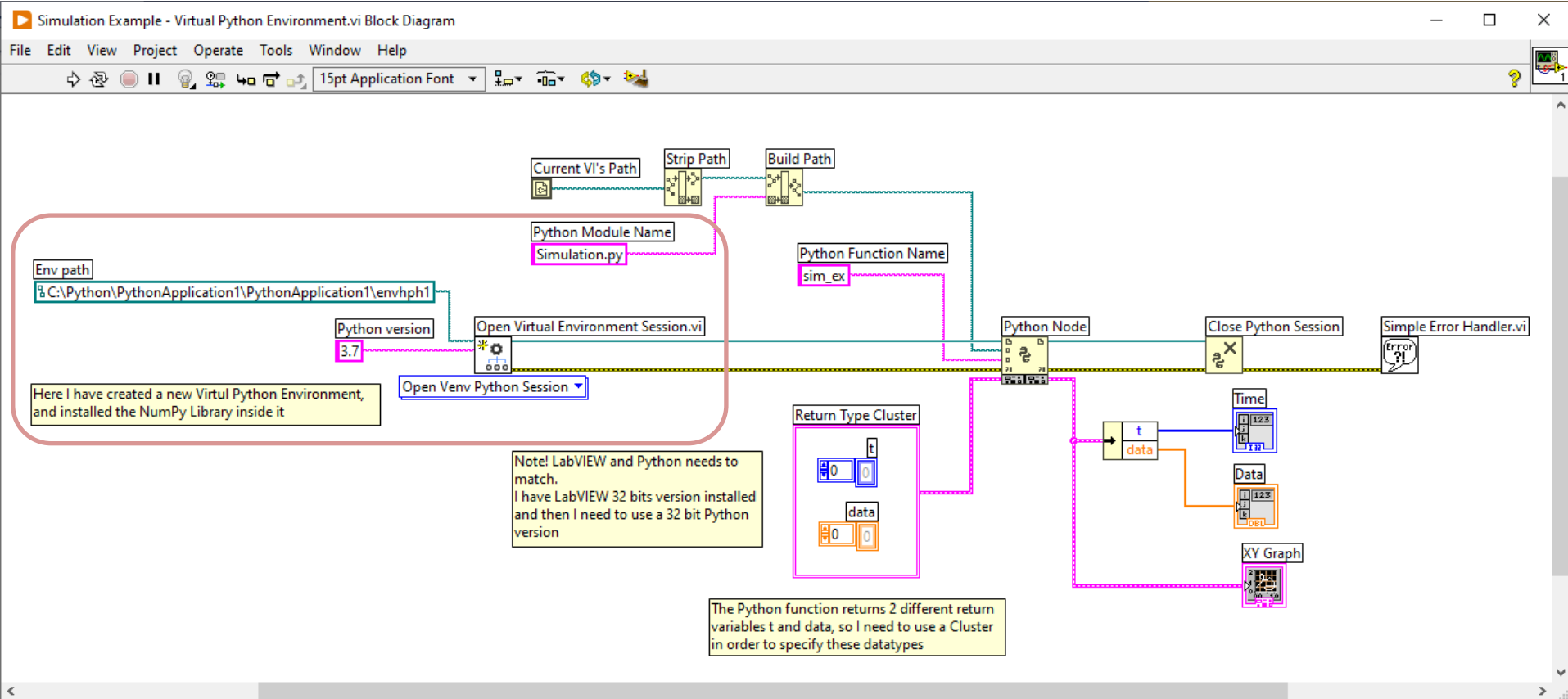
Properties

envhph1 (Python 3.7 (32-bit)) Environment Properties

Folder Name	envhph1
Full Path	C:\Python\PythonApplication1\PythonApplication1\envhph1\
Version	3.7

Folder Name
Name of this folder

Virtual Python Environment



<https://www.halvorsen.blog>

MATLAB Integration



Hans-Petter Halvorsen

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MATLAB Integration

Connectivity

↑ Search Customize

Alternative 2:

Libraries & Executables

.NET

Python

Input Device Control

ActiveX

Database

Web Services

Windows Registry Acces...

DataFinder

MATLAB(R)

MATLAB(R)

↑ Search Customize

Open MATLAB Session

Call MATLAB Function

Close MATLAB Session

Alternative 1:

Mathematics

↑ Search Customize

Numeric

Elementary

Linear Algebra

Fitting

Interp & Extrap

Integ & Diff

Prob & Stat

Optimization

Differential Eqs

Geometry

Polynomial

Script & Formula

Scripts & Formulas

↑ Search Customize

Formula Node

Script Nodes

Formula

Formula Parsing

Expression Node

1D & 2D Evaluation

Calculus

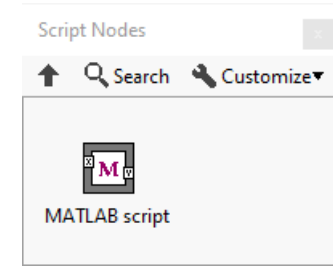
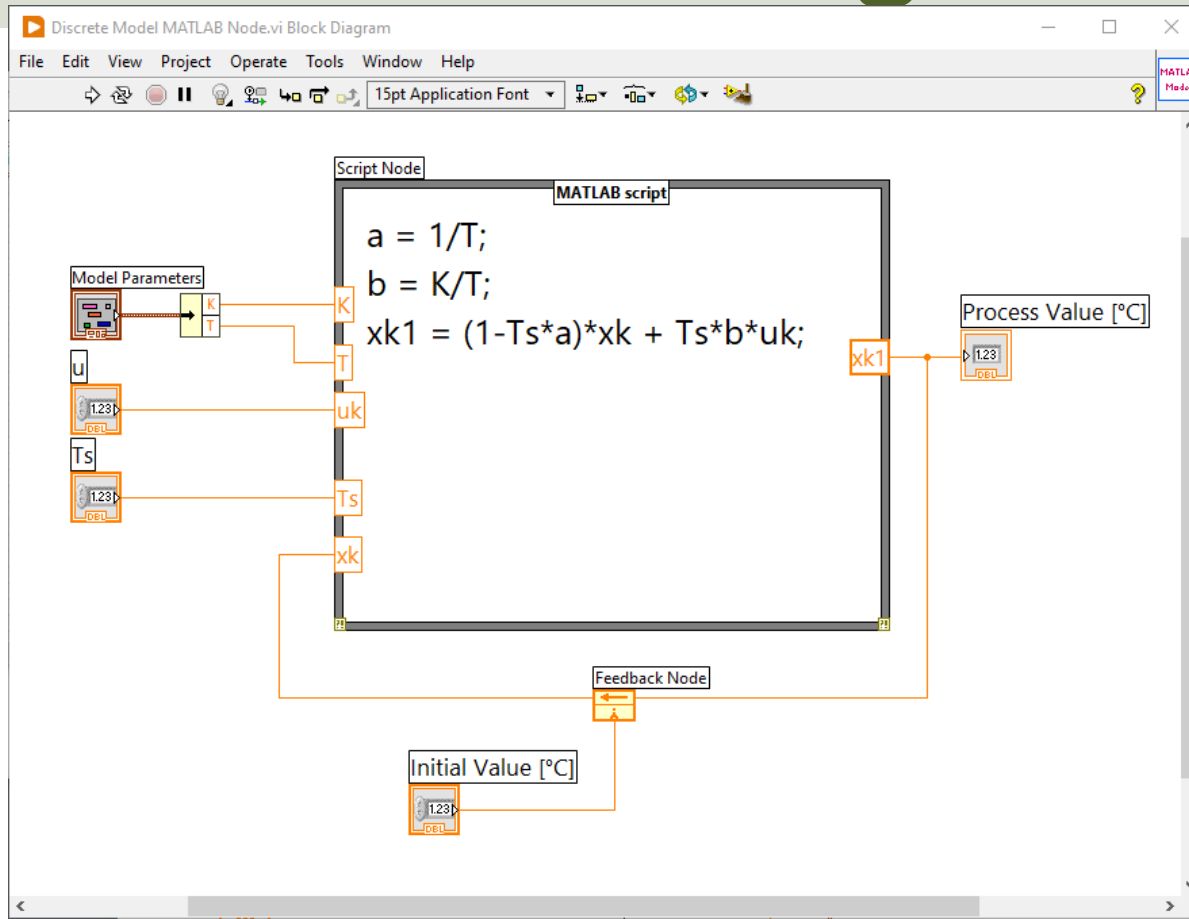
Zeros

Script Nodes

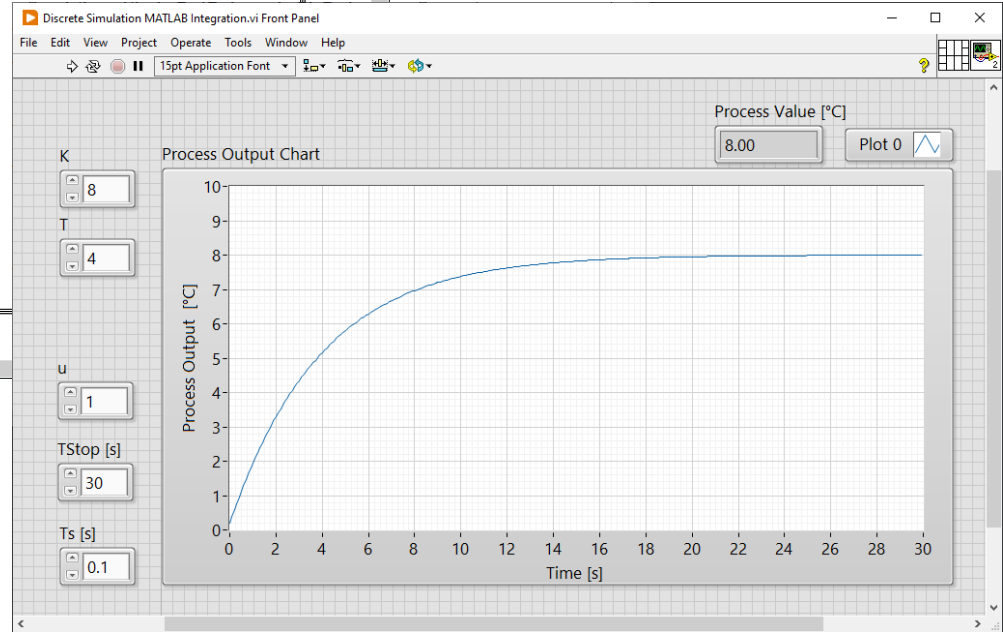
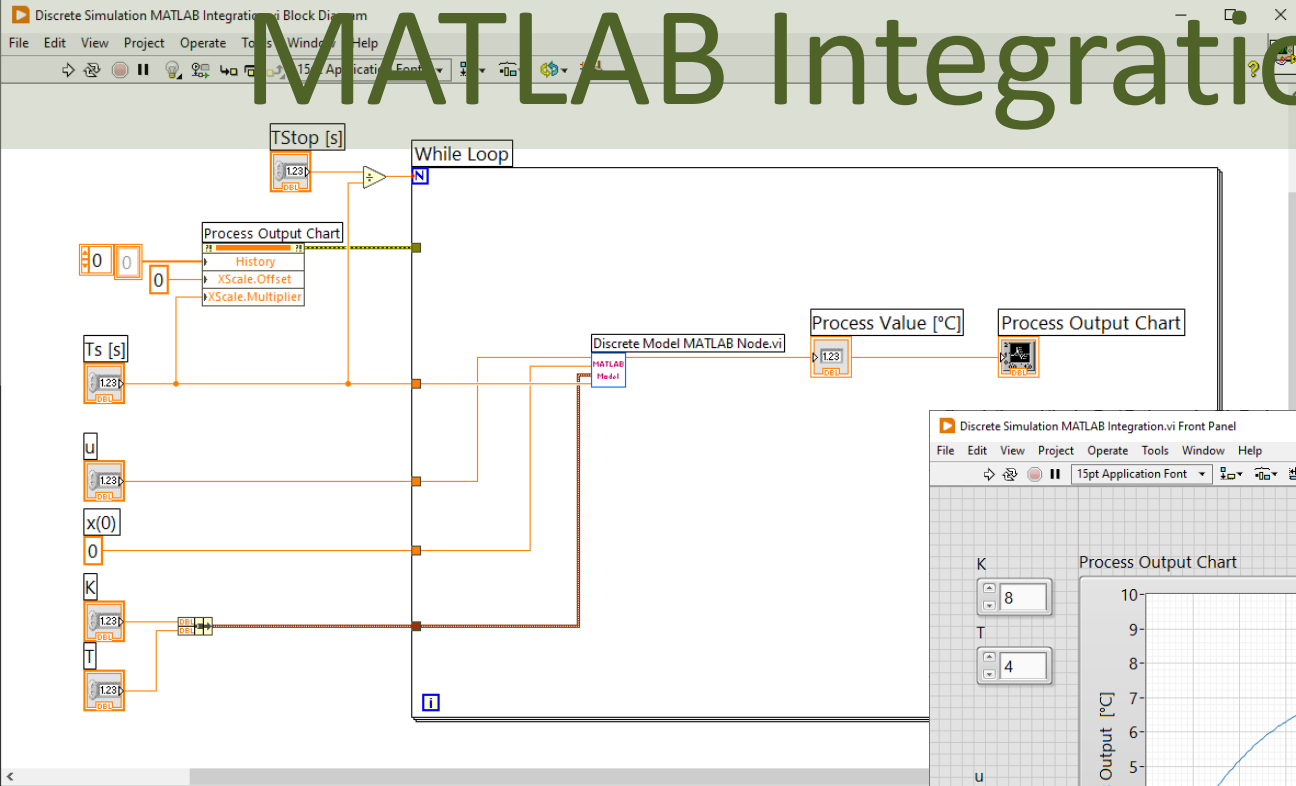
↑ Search Customize

MATLAB script

MATLAB Integration Alt1



MATLAB Integration Alt1



MATLAB Integration Alt2

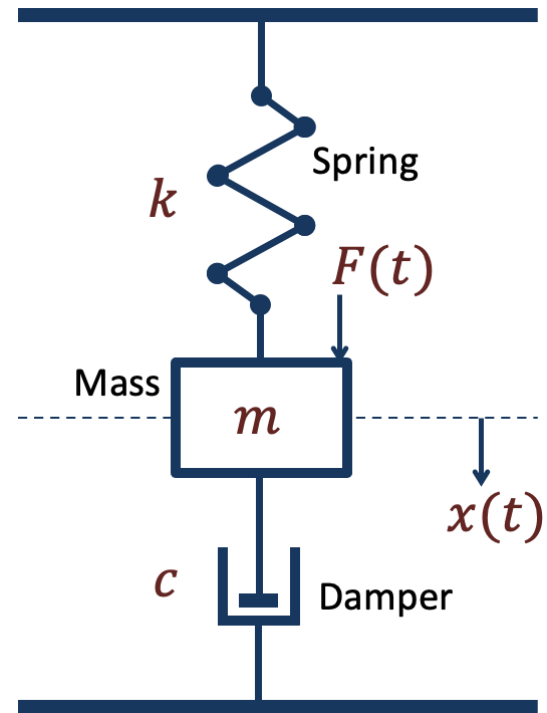
```
function [t, x1, x2] = mass_spring_damper_system()

% Simulation of Mass-Spring-Damper System
clear
clc

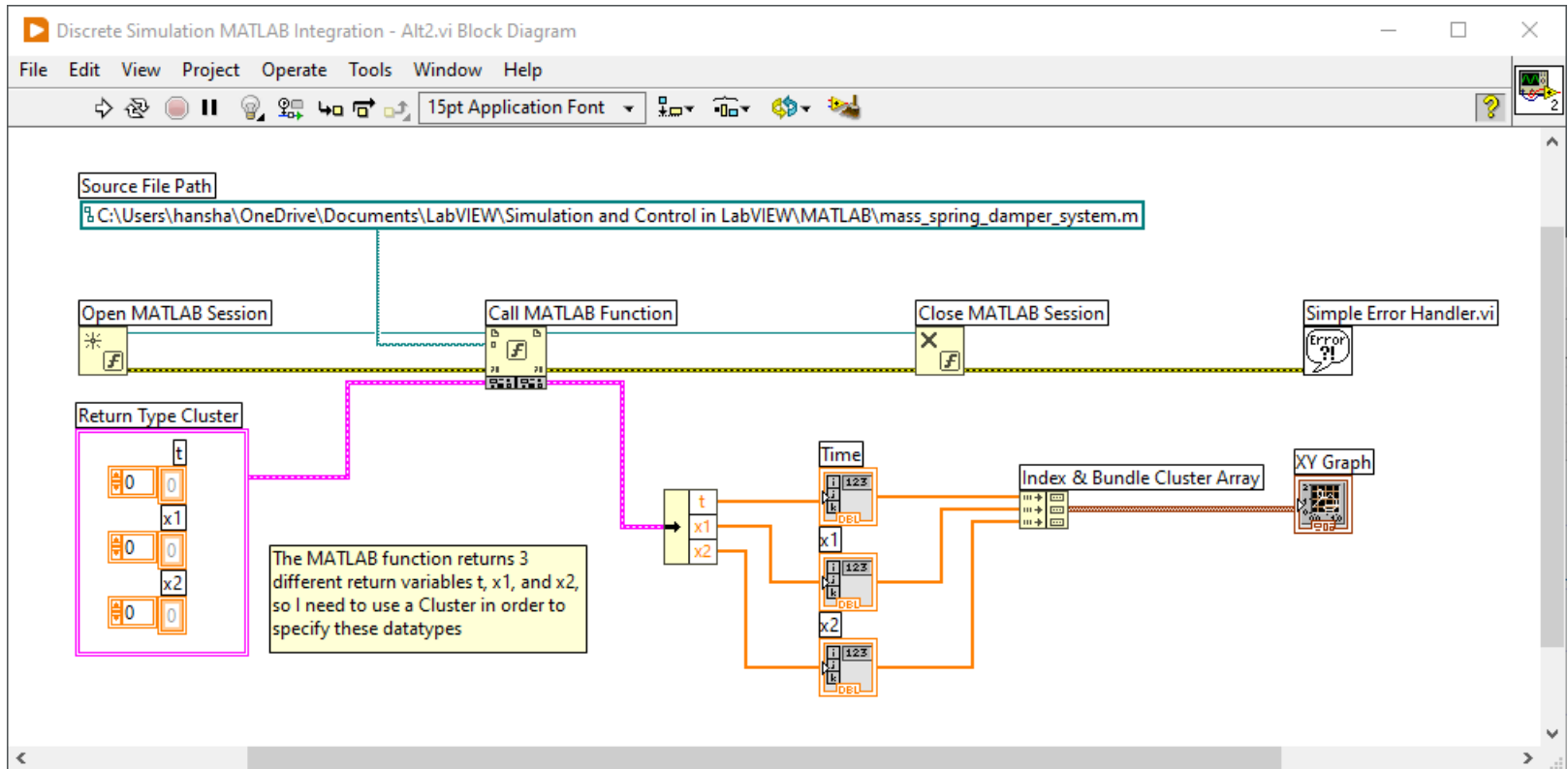
% Model Parameters
c = 4; % Damping constant
k_stiff = 2; % Stiffness of the spring
m = 20; % Mass
F = 5; % Force

% Simulation Parameters
Ts = 0.1;
Tstart = 0;
Tstop = 60;
N = (Tstop-Tstart)/Ts; % Simulation length
t = Tstart : Ts : Tstop;
x1 = zeros(N,1);
x2 = zeros(N,1);
x1(1) = 0; % Initial Position
x2(1) = 0; % Initial Speed

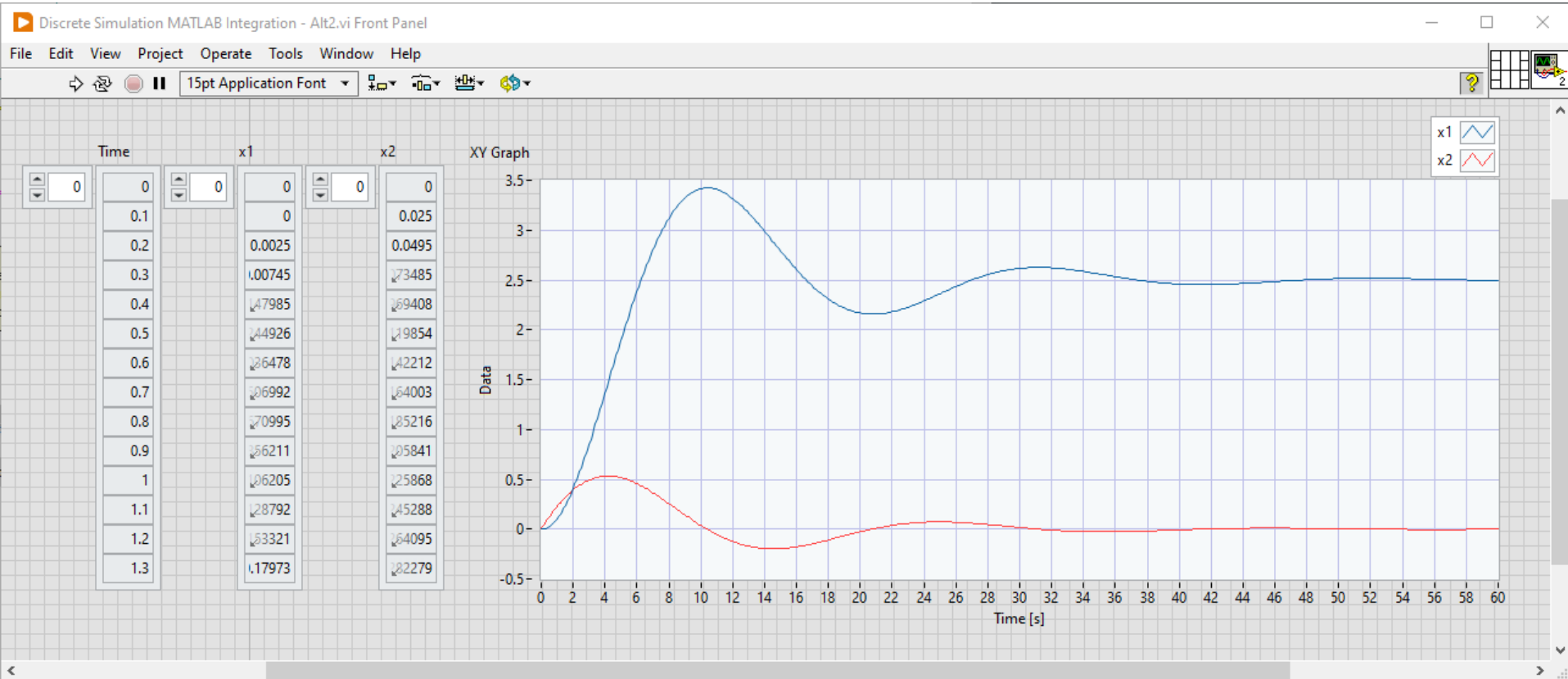
% Simulation
for k=1:N
    x1(k+1) = x1(k) + Ts * x2(k);
    x2(k+1) = (-Ts*k_stiff)/m * x1(k) + (1 - (Ts*c)/m) * x2(k) + (Ts/m) * F;
end
```



MATLAB Integration Alt2



MATLAB Integration Alt2



MATLAB Alt2 - Improved

- In the Alt2 Example we just got the results from the Simulation
- Let's also make it possible to set the Model Parameters, etc. from LabVIEW
- This will then be sent as arguments to the MATLAB Function

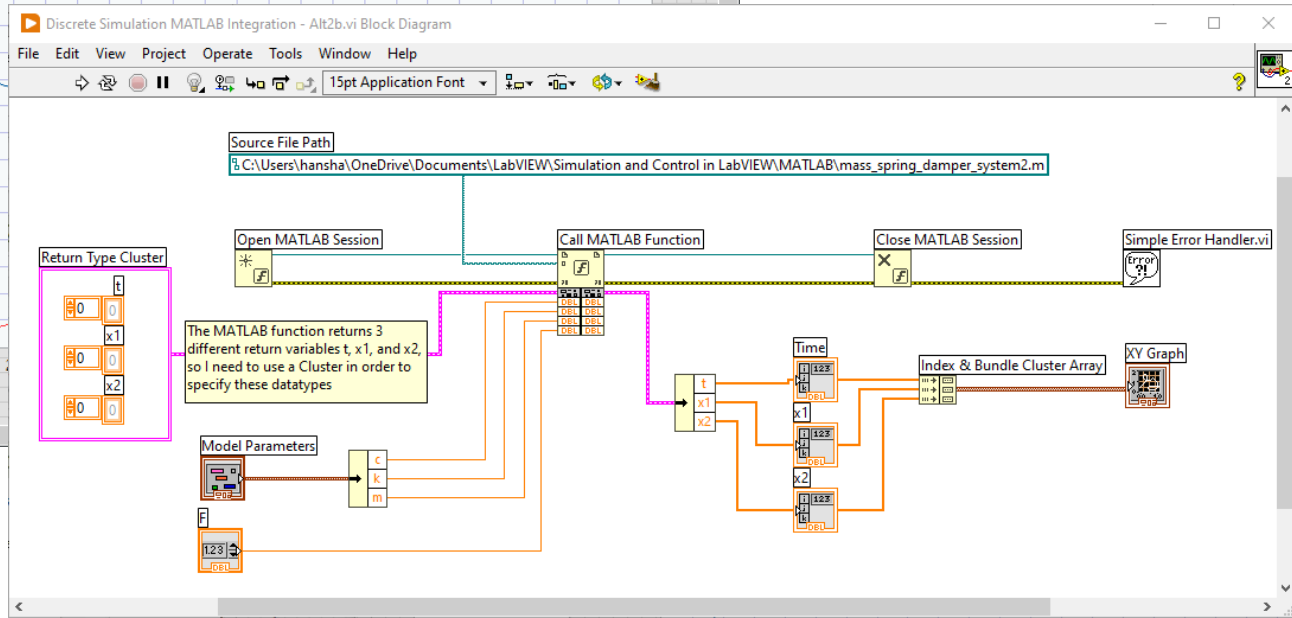
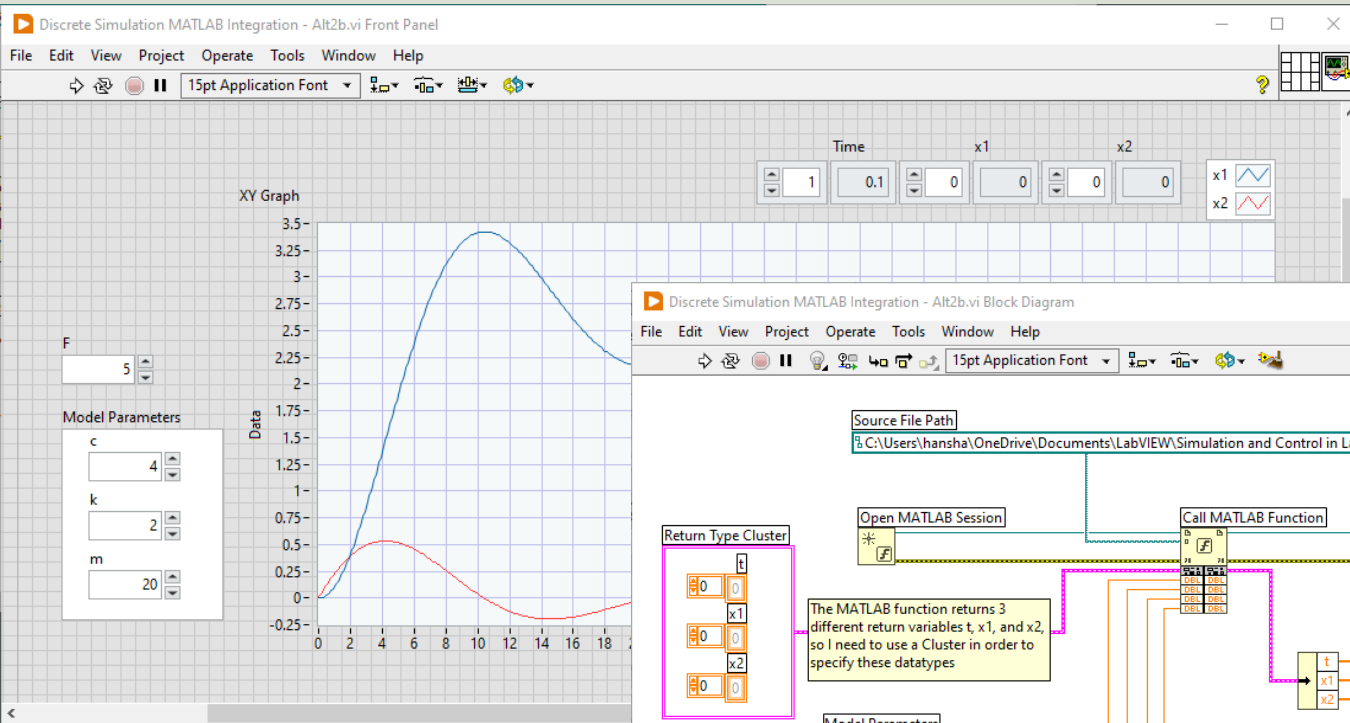
MATLAB Alt2 - Improved

```
function [t, x1, x2] = mass_spring_damper_system2(c, k_stiff, m, F)
% Simulation of Mass-Spring-Damper System

% Simulation Parameters
Ts = 0.1;
Tstart = 0;
Tstop = 60;
N = (Tstop-Tstart)/Ts; % Simulation length
t = Tstart : Ts : Tstop;
x1 = zeros(N,1);
x2 = zeros(N,1);
x1(1) = 0; % Initial Position
x2(1) = 0; % Initial Speed

% Simulation
for k=1:N
    x1(k+1) = x1(k) + Ts * x2(k);
    x2(k+1) = -(Ts*k_stiff)/m * x1(k) + (1 - (Ts*c)/m) * x2(k) + (Ts/m) * F;
end
```

MATLAB Alt2 - Improved



<https://www.halvorsen.blog>

“Discrete Integrator”

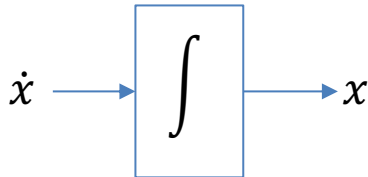


Hans-Petter Halvorsen

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“Discrete Integrator”

- In previous examples we needed to first find a discrete version of our differential equation(s) using Euler or other discretization methods
- It can be time-consuming and cumbersome to find these discrete differential equations
- So, we may want to create a “Discrete Integrator” and in that way we don’t need to solve or find discrete versions of the differential equation(s)



Integrator

Assume a general Differential Equation:

$$\dot{x} = f(t, x)$$

The purpose is to find x .

So, to find x we can Integrate $f(t, x)$:

$$x = \int f(t, x)$$

Discrete Integrator

Given

$$\dot{x} = f(t, x)$$

We use Euler to find a discrete version

$$\dot{x} \approx \frac{x(k+1) - x(k)}{T_s}$$

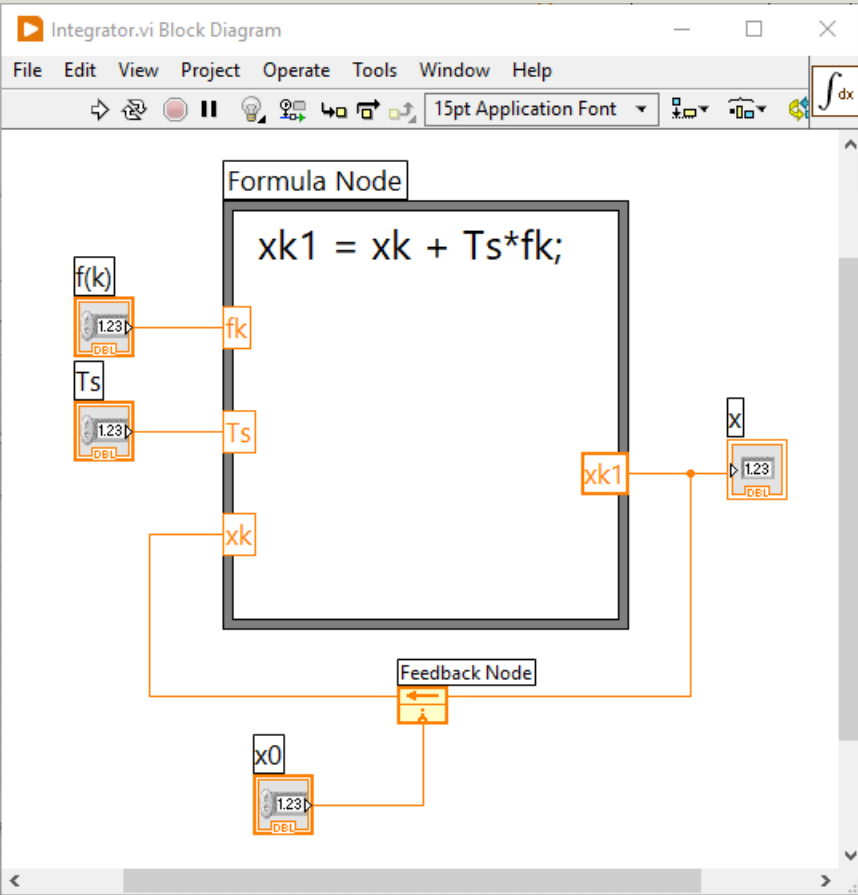
Then we get:

$$\frac{x(k+1) - x(k)}{T_s} = f(k)$$

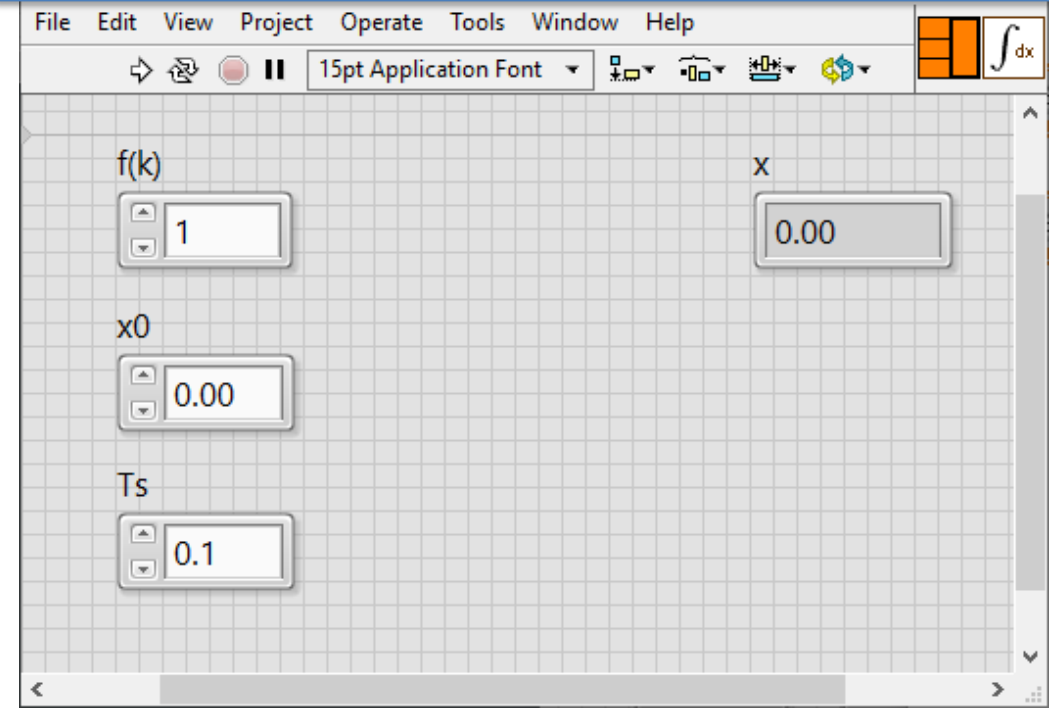
Which gives:

$$x(k+1) = x(k) + T_s f(k)$$

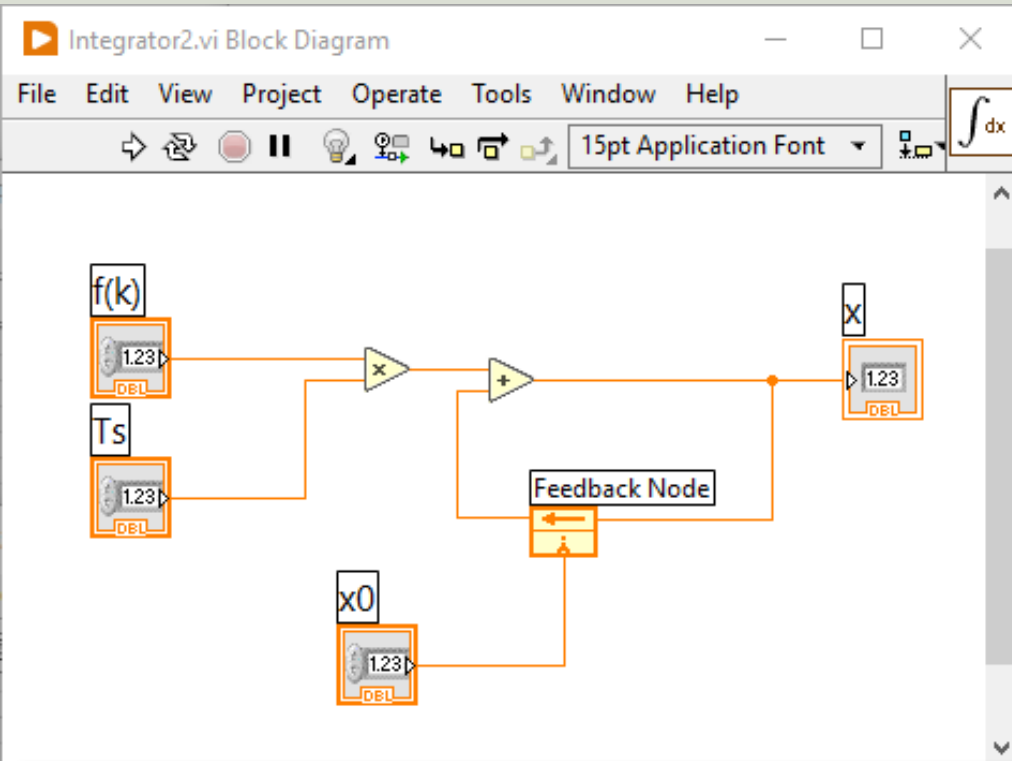
LabVIEW Integrator



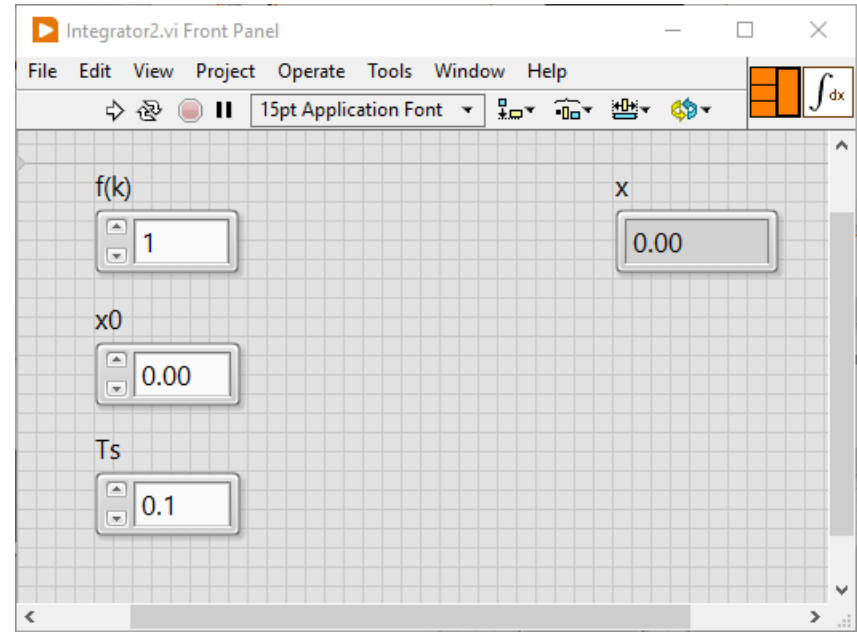
Here is the LabVIEW implementation of our Integrator. Basically, we can use this for all kind of differential systems, either we have one or many differential equations. Here is a Formula Node used, but you could have used pure LabVIEW code as well



LabVIEW Integrator – Alt2



Here we have used pure LabVIEW code instead of a Formula Node



We can also improve the Integrator block by implementing Saturation, i.e., include inputs for Lower Limit and Upper Limit

Simulation Example

We have the general differential equation:

$$\dot{x} = f(t, x)$$

Then

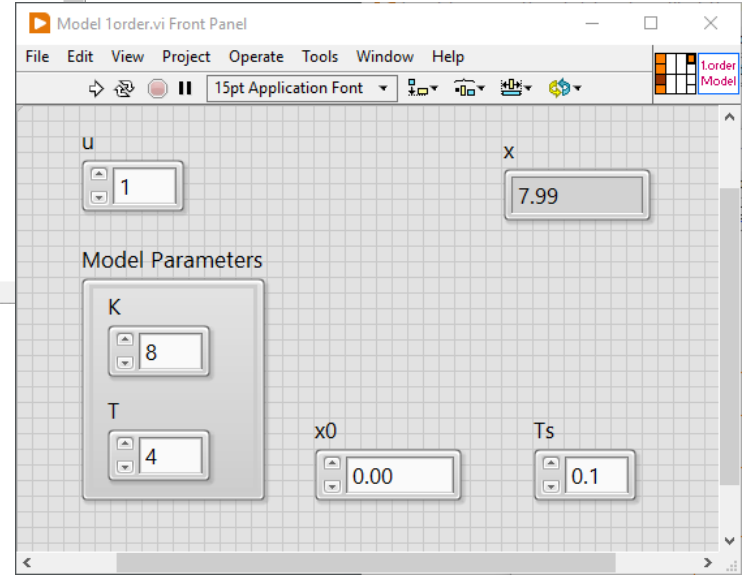
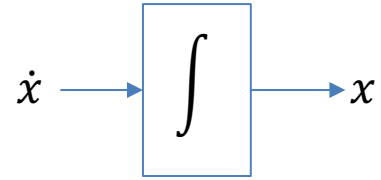
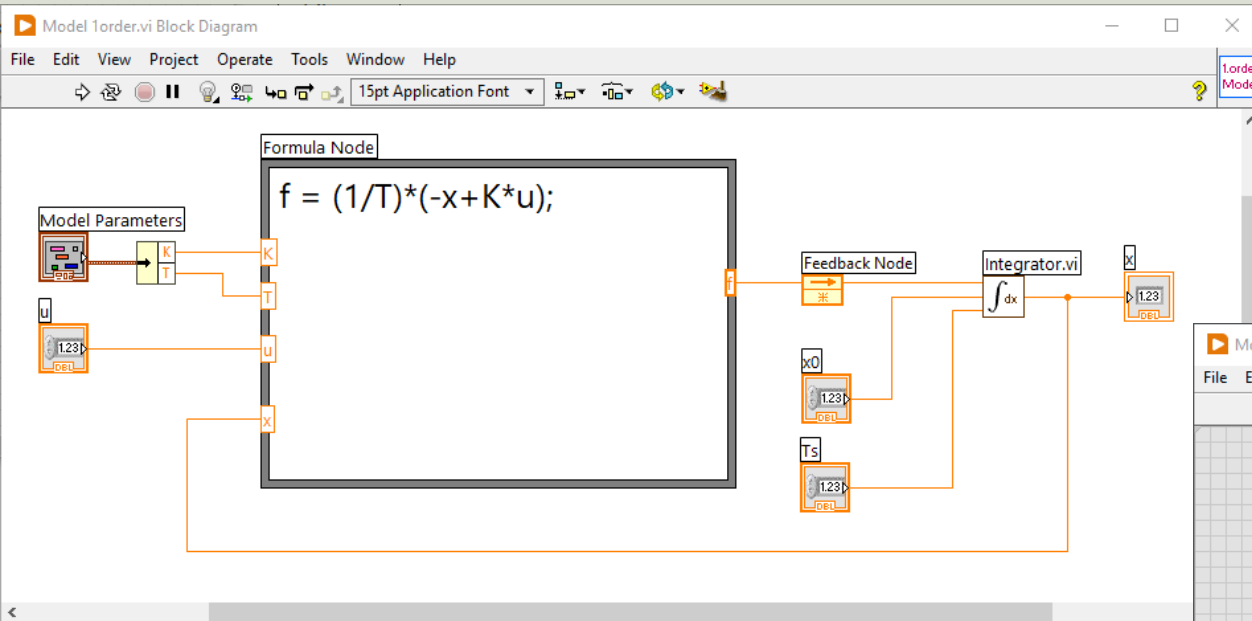
$$x = \int f(t, x)$$

Let's test out this discrete Integrator on our standard 1.order system:

$$\dot{x} = \frac{1}{T} (-x + Ku)$$

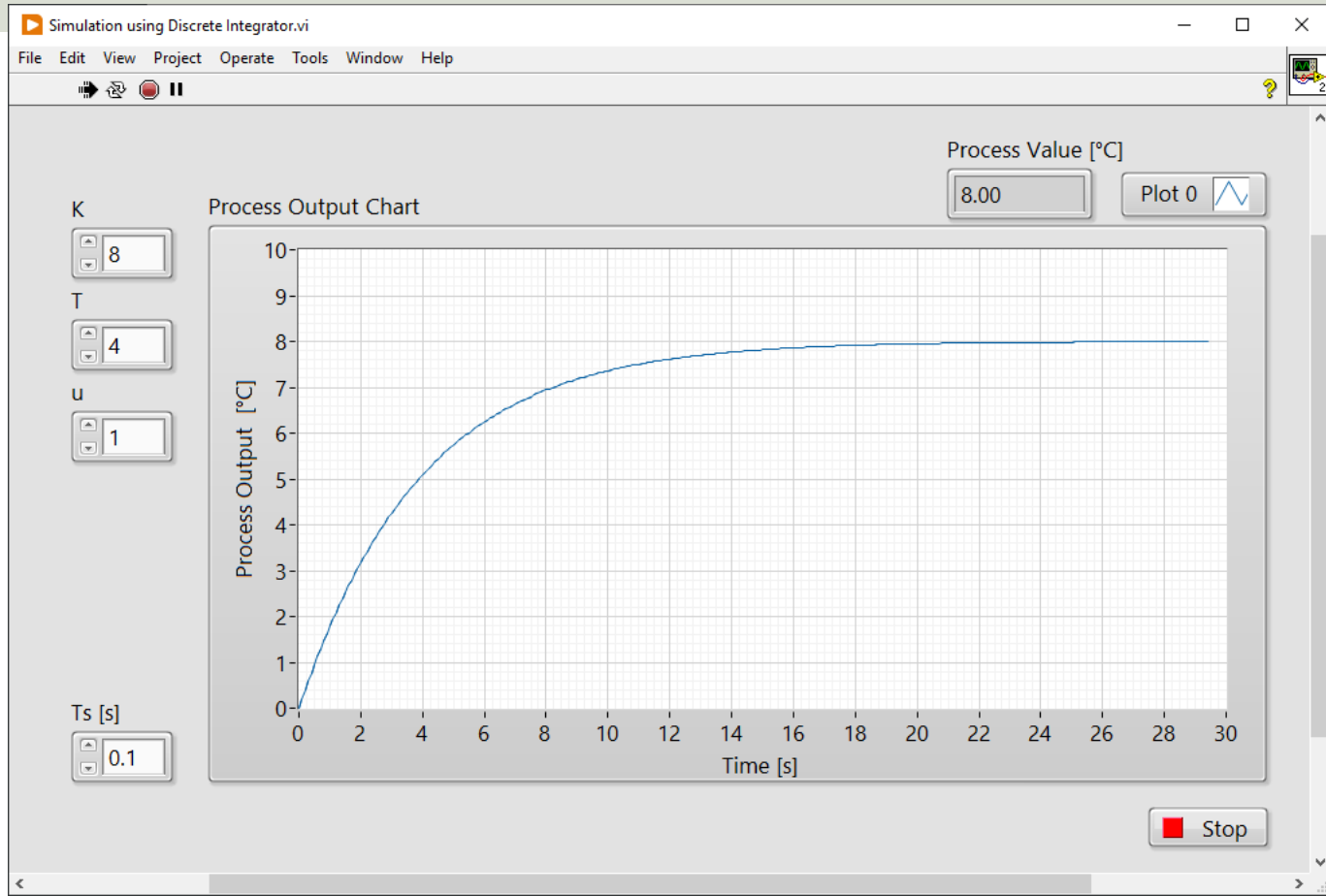
$$\dot{x} = \frac{1}{T}(-x + Ku)$$

LabVIEW



Now we only need to implement the right side of the differential equation. Then we use our new “Discrete Integrator” to find the solution in each iteration

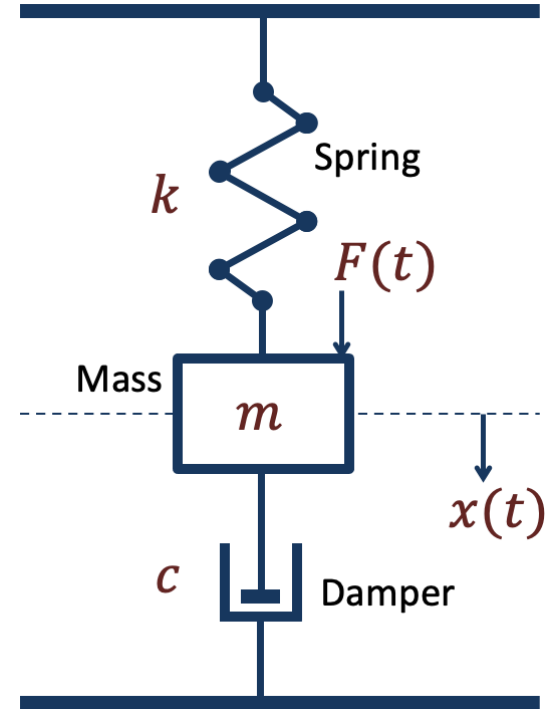
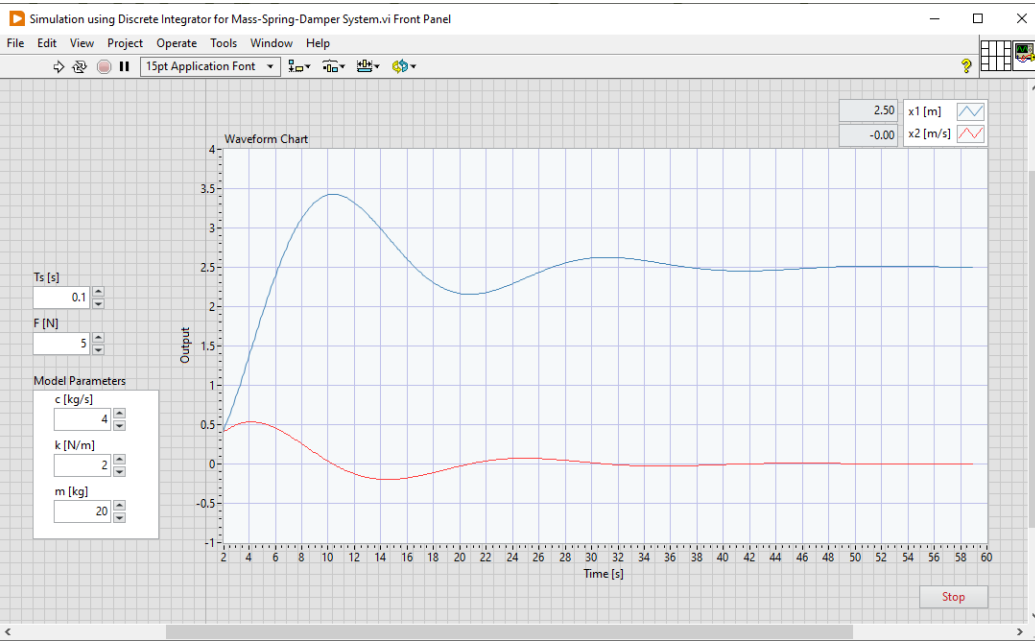
LabVIEW



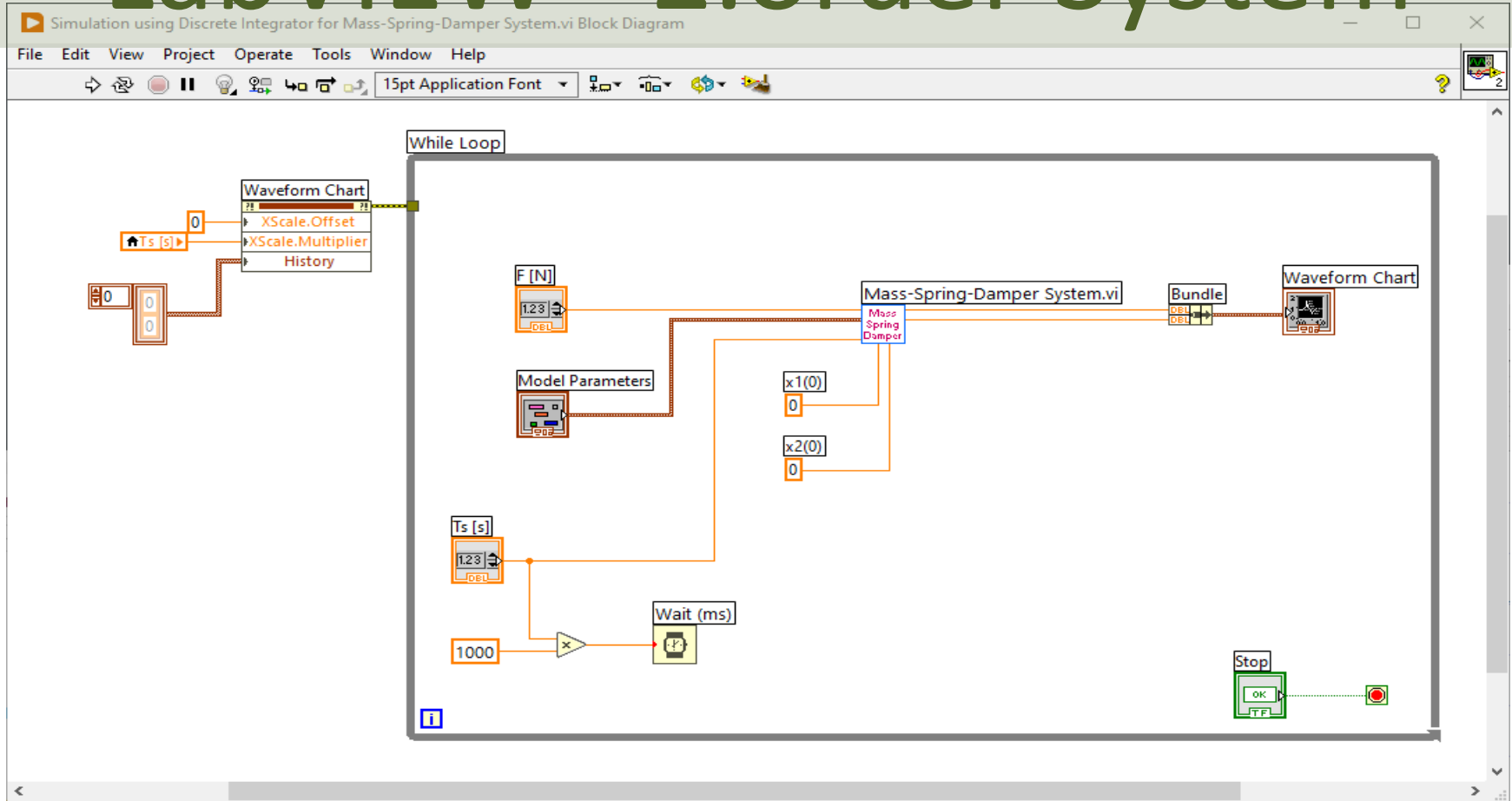
2.order System

Let's test out new Integrator block on a 2.order system. We can use the previous Mass-Spring-Damper System:

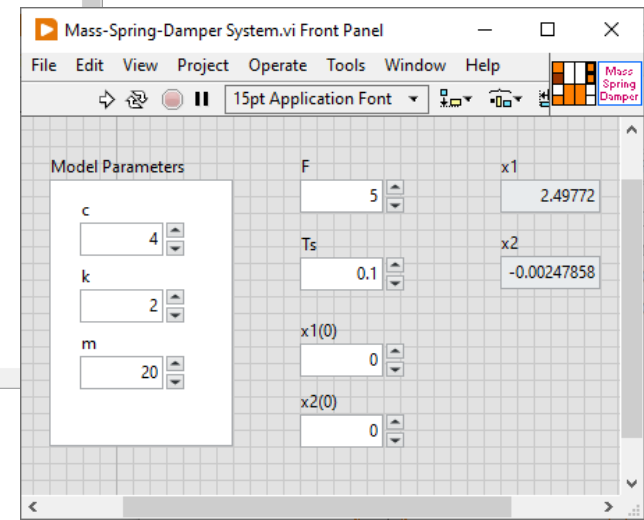
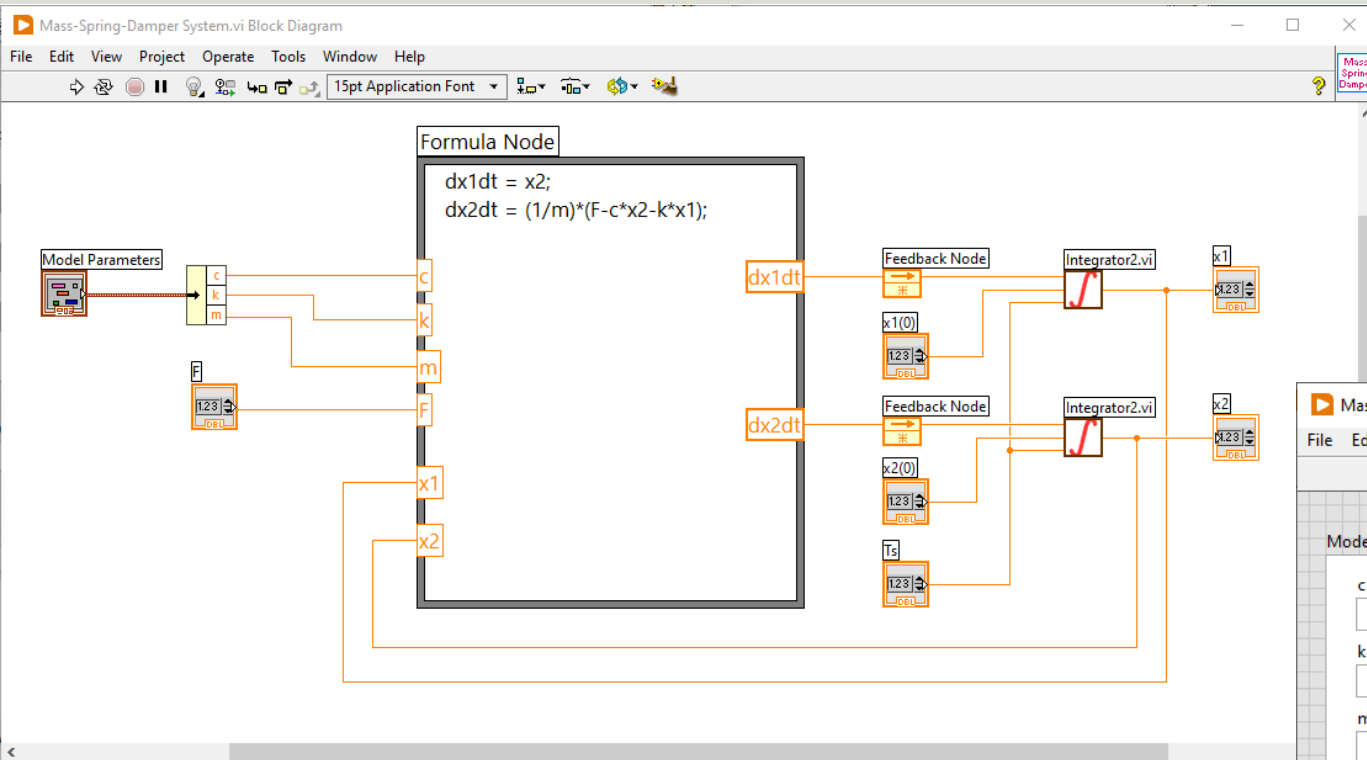
$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = \frac{1}{m}(F - cx_2 - kx_1)$$



LabVIEW - 2.order System



LabVIEW - 2.order System



It is also possible to implement this in a more general way, i.e., create our own “ODE Solver.vi” that we used in one of the previous examples

<https://www.halvorsen.blog>

1.order System with Time Delay



Hans-Petter Halvorsen

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Time Delay

The equation for a Time Delay can be written as:

$$y = u(t - \tau)$$

Laplace:

$$y(s) = u(s)e^{-\tau s}$$

This gives the following Transfer function:

$$H(s) = \frac{y(s)}{u(s)} = e^{-\tau s}$$

Laplace Transformation pairs:

$$\dot{x} \Leftrightarrow sx(s)$$

$$u(t - \tau) \Leftrightarrow u(s)e^{-\tau s}$$

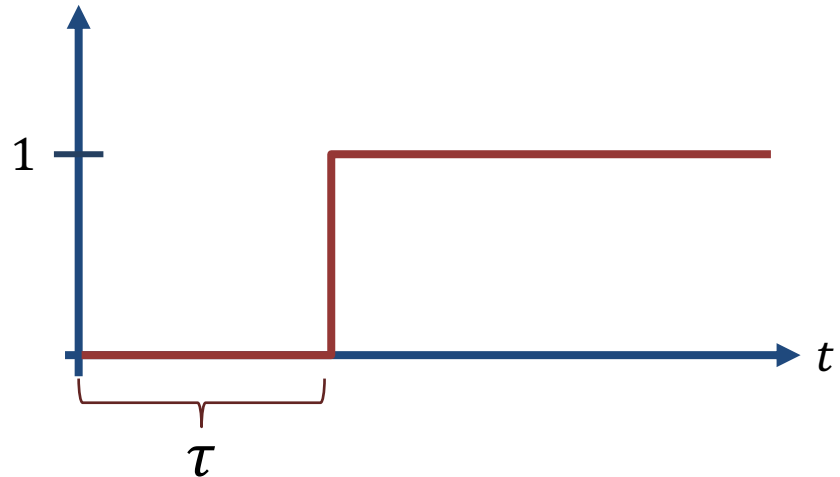
τ is the Time Delay

Time Delay

Transfer Function for Time Delay:

$$H(s) = e^{-\tau s}$$

Step Response for a Time Delay:



τ is the Time Delay in seconds

1.order System with Time Delay

A general 1. order System with Time Delay can be written as:

$$\dot{x} = -ax + bu(t - \tau)$$

$$\text{Where } a = \frac{1}{T} \text{ and } b = \frac{K}{T}$$

It can also be written like this:

$$\dot{x} = \frac{1}{T} [-x + Ku(t - \tau)]$$

Where K is the Gain, T is the Time constant and τ is the Time Delay

Transfer Function

Transfer Function for Time Delay:

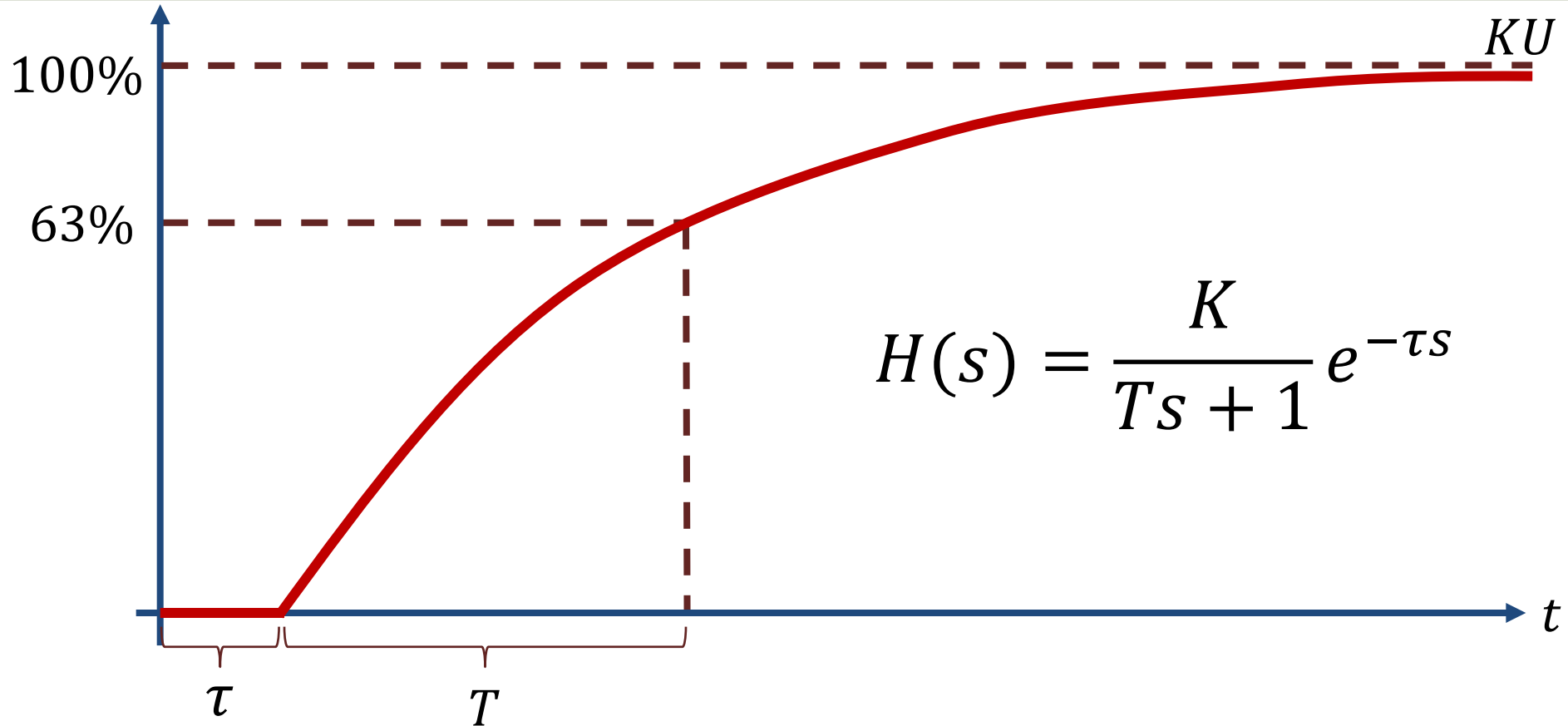
$$H(s) = \frac{y(s)}{u(s)} = e^{-\tau s}$$

1.order Transfer Function with Time Delay:

$$H(s) = \frac{y(s)}{u(s)} = \frac{K}{Ts + 1} e^{-\tau s}$$

Where K is the Gain, T is the Time constant and τ is the Time Delay

Step Response



Discrete Time Delay Function

Let's create a Discrete Time Delay Function in LabVIEW

The equation for a Time Delay can be written as:

$$y = u(t - \tau)$$

Discrete version:

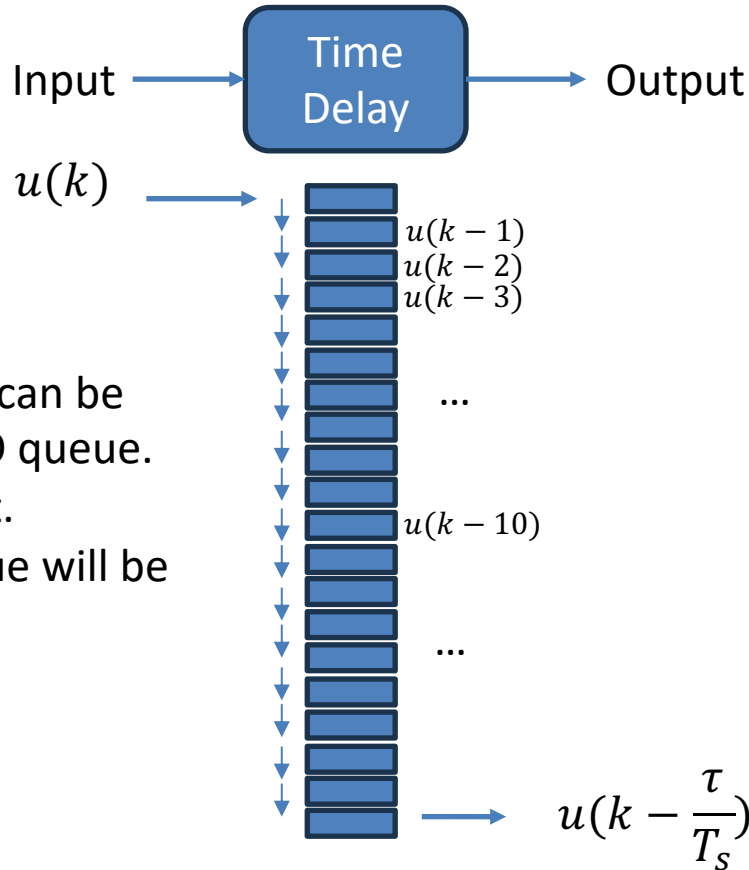
$$y(k) = u\left(k - \frac{\tau}{T_s}\right)$$



Assuming, e.g., $\tau = 2s$ and $T_s = 0.1s$ we get $u(k - 20)$

This means we must remember the 20 previous samples of $u(k)$ in our calculations

Discrete Time Delay Function



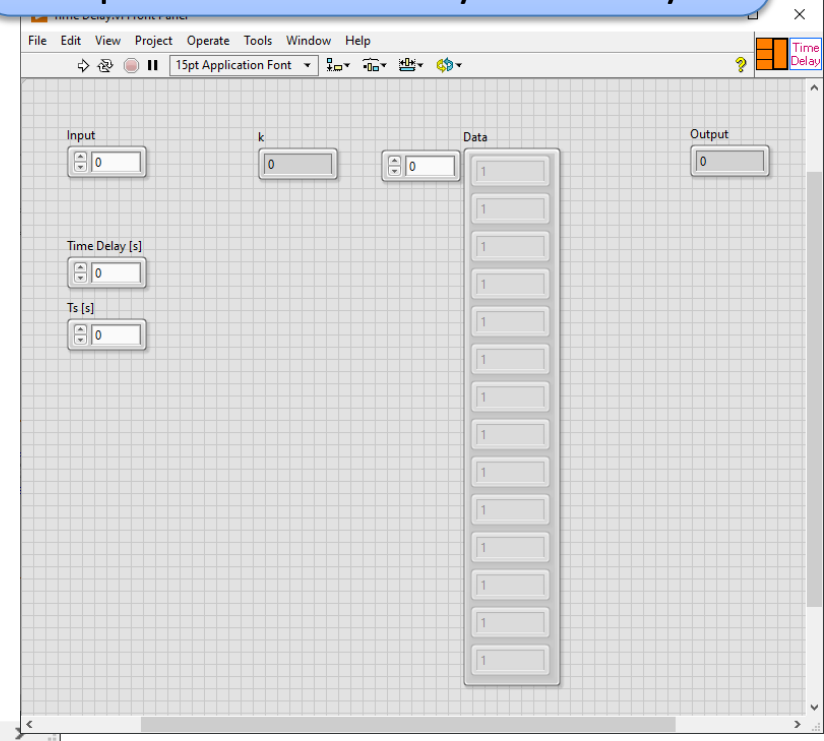
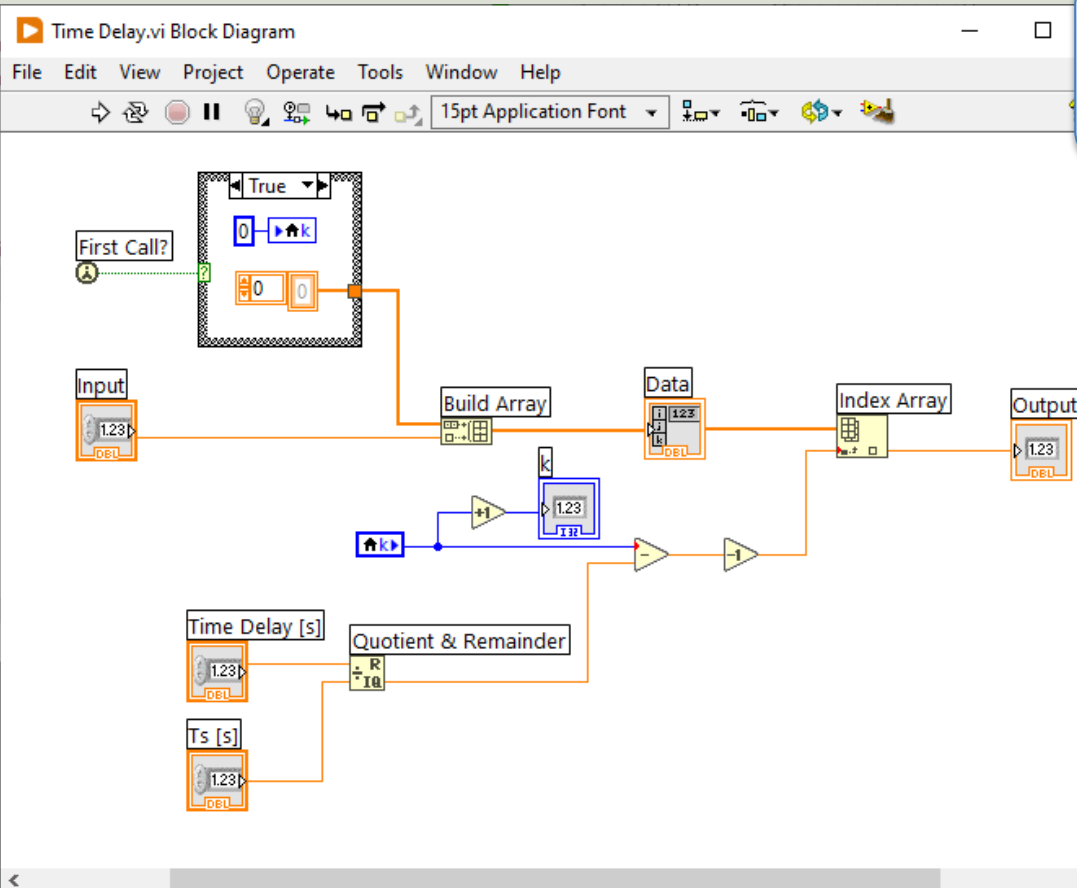
A Discrete Time Delay can be implemented as a FIFO queue.
FIFO – First In First Out.
The length of the queue will be

$$N = \frac{\tau}{T_s}$$

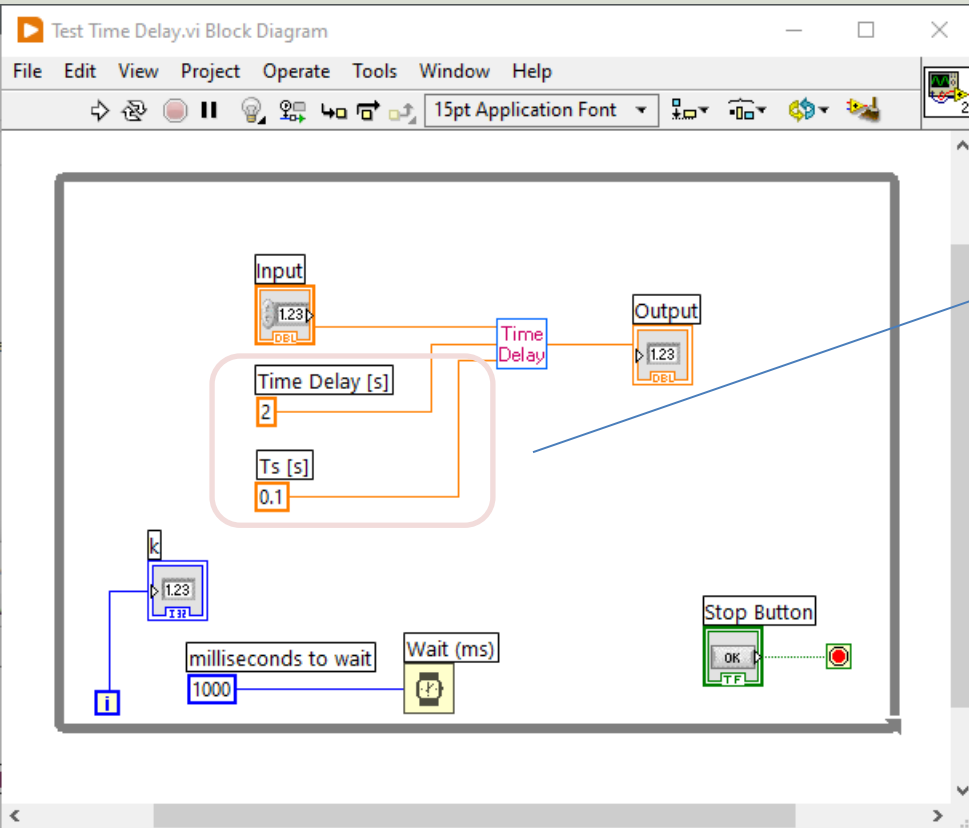
E.g.,
 $\tau = 2s$ and $T_s = 0.1s$
Then we get $u(k - 20)$

LabVIEW Time Delay Function

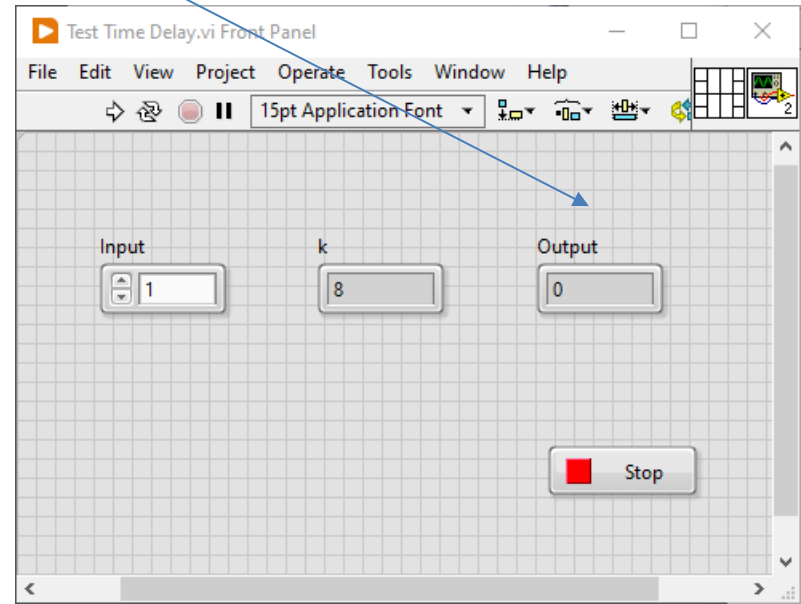
This is one way to implement a Time Delay in LabVIEW, but it can also be implemented in many other ways



Test Time Delay Function



When $k=21$ the Output will be 1 in this case



Discretization

We have the continuous differential equation: $\dot{x} = -ax + bu(t - \tau)$

We apply **Euler**: $\dot{x} \approx \frac{x(k+1) - x(k)}{T_s}$

Where $a = \frac{1}{T}$ and $b = \frac{K}{T}$

Then we get:

The discrete version of τ is $\frac{\tau}{T_s}$

$$\frac{x(k+1) - x(k)}{T_s} = -ax(k) + bu(k - \frac{\tau}{T_s})$$

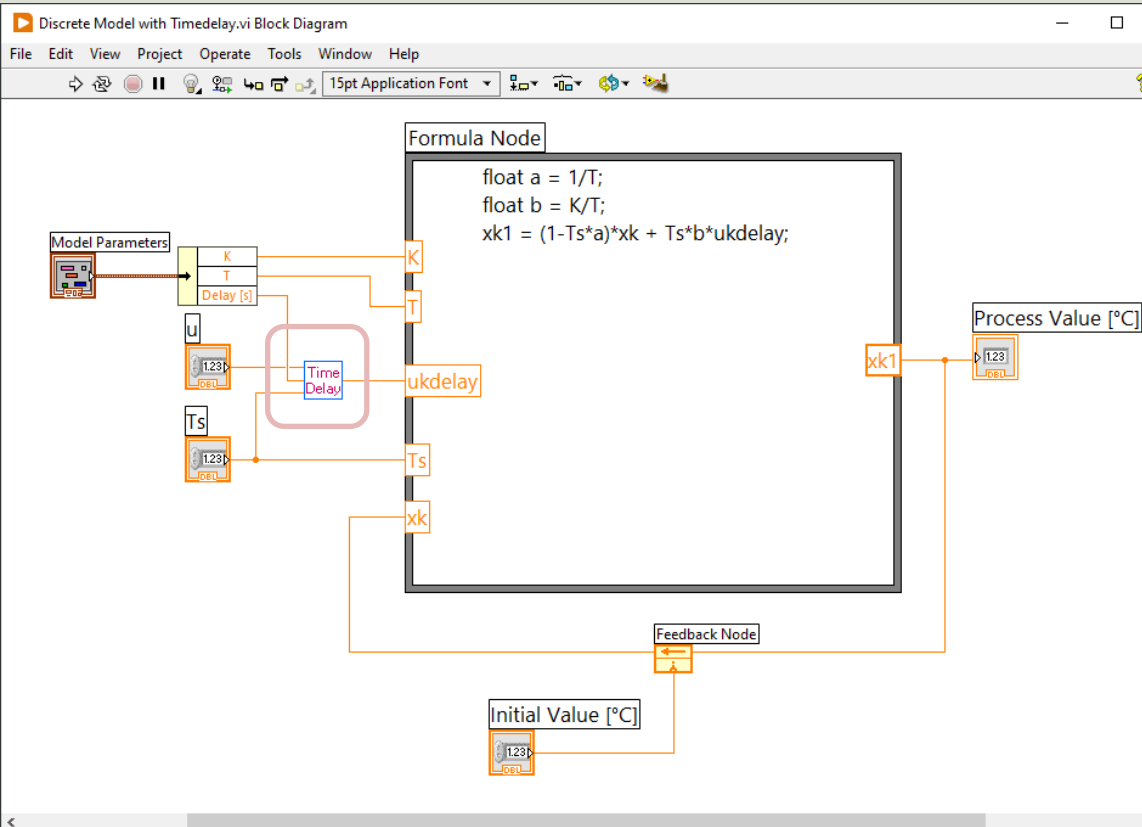
This gives the following discrete differential equation (difference equation):

$$x(k+1) = (1 - T_s a)x(k) + T_s bu(k - \frac{\tau}{T_s})$$

Assuming $\tau = 2s$ and $T_s = 0.1s$ we get $u(k - 20)$

This means we must remember the 20 previous samples of $u(k)$

LabVIEW

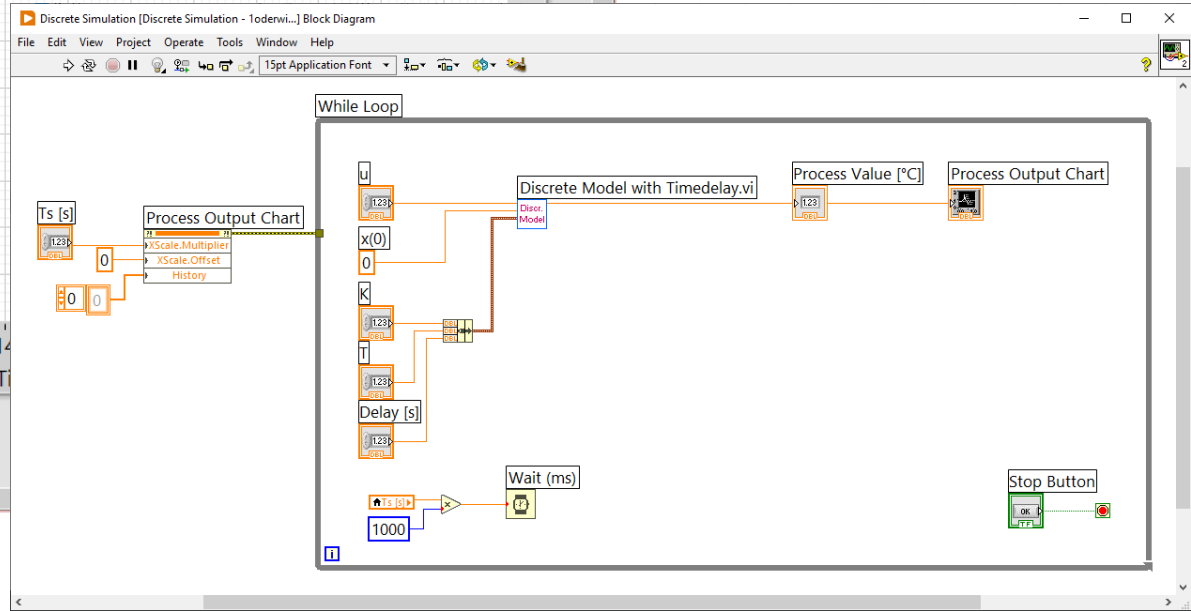
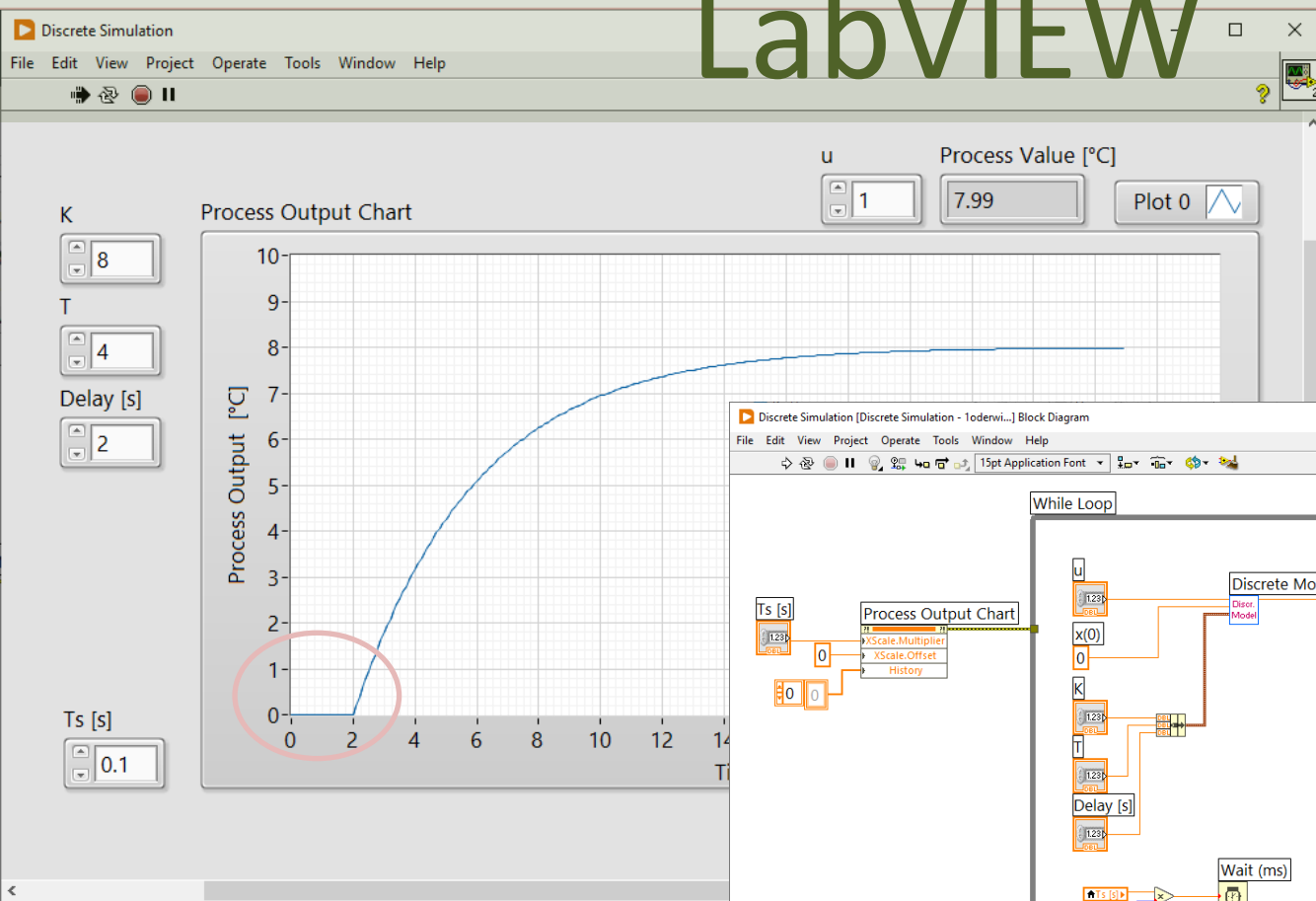


Discrete Model with Timedelay.vi Front Panel

The front panel provides a graphical interface for configuring and monitoring the model. It includes the following controls:

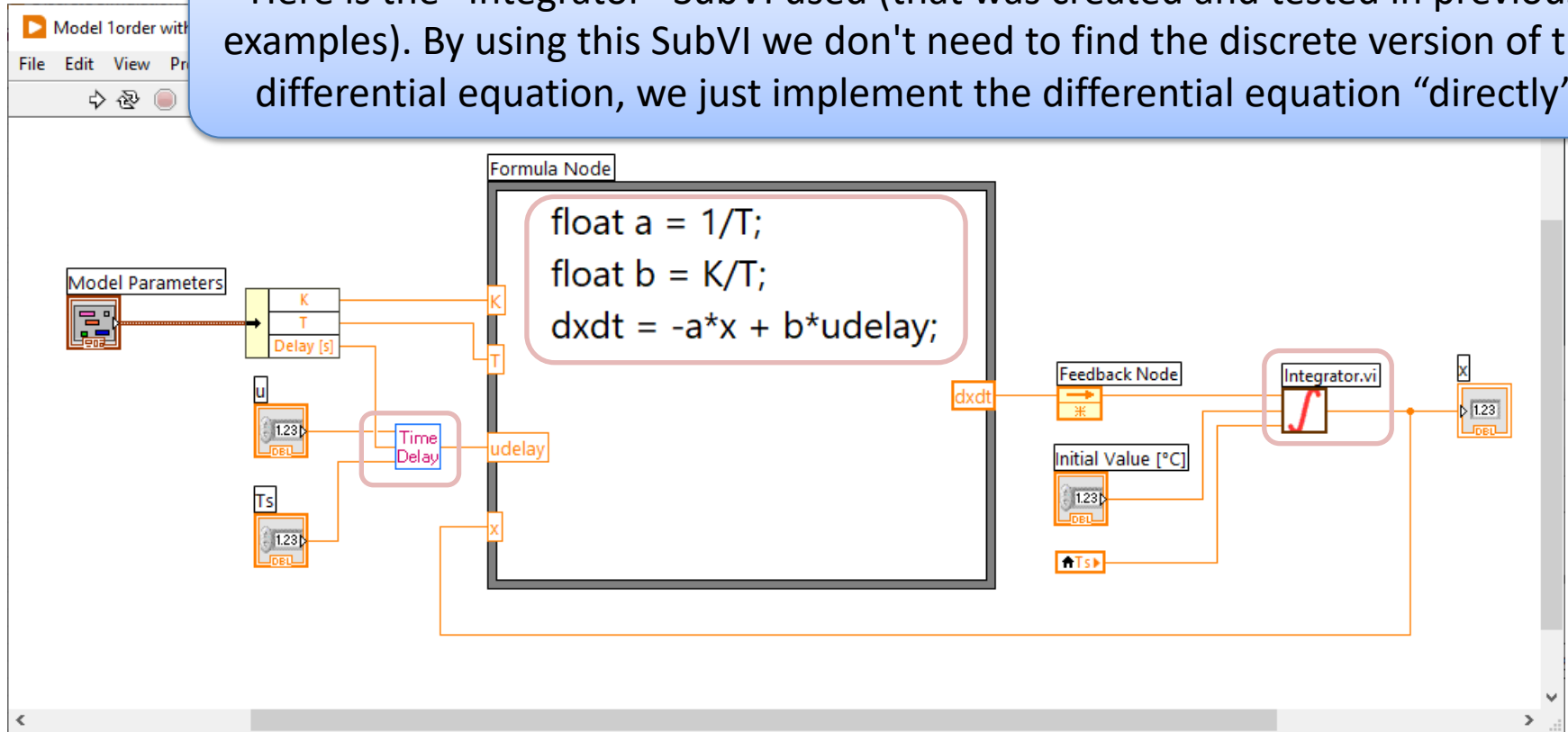
- U:** A numeric control with a value of 1.
- Initial Value [°C]:** A numeric control with a value of 0.00.
- Model Parameters:** A cluster of numeric controls for K (value 1), T (value 4), and Delay [s] (value 2).
- Ts:** A numeric control with a value of 0.1.
- Process Value [°C]:** A numeric display showing the current value of 0.00.

LabVIEW



Alternative Model Implementation

Here is the “Integrator” SubVI used (that was created and tested in previous examples). By using this SubVI we don't need to find the discrete version of the differential equation, we just implement the differential equation “directly”



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